

Bab 3: Sistem Diskrit

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Notasi Sinyal Diskrit

- Simbol sinyal diskrit : $x(n)$ atau $x(k)$; $y(n)$ atau $y(k)$, $h(n)$ atau $h(k)$
- Pergeseran:
 $x(n-1)$ menyatakan sampel ke $(n-1)T$, dengan T adalah perioda sampling
- Deret diskrit Impuls

$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n-k)$$

$$\text{Unit impulse : } \delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Unit step

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

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Format Sinyal Diskrit

- Dalam bentuk fungsi

$$x(n) = \begin{cases} 1 & n = \text{integer ganjil} \\ 0 & n = \text{integer genap} \end{cases}$$

- Dalam bentuk tabel

n	... -2 -1 0 1 2 3 4 ...
X(n)	... 0 1 2 3 2 1 1 ..

- Dalam bentuk deret

$$x(n) = \{0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6\}$$

$$x(n-2) = \{0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6\}$$

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Sinyal Sinus

Bentuk analog sinyal sinus :

$$x(t) = A \sin(\Omega t + \phi) = A \sin(2\pi f t + \phi), \quad \Omega = 2\pi f$$

Bentuk digital sinyal sinus :

$$\begin{aligned} x(n) &= A \sin(\Omega n T + \phi) = A \sin(2\pi f n T + \phi) \\ &= A \sin(\omega n + \phi) = A \sin(\pi F n + \phi), \end{aligned}$$

$$\text{Frekuensi Digital : } \omega = \Omega T = \frac{2\pi f}{f_s}$$

$$\text{Frekuensi Digital ternormalisasi : } F = \frac{\omega}{\pi} = \frac{f}{(f_s/2)}$$

Table 3.1 Units, relationships, and ranges of four frequency variables

Variables	Unit	Relationship	Range
Ω	Radians per second	$\Omega = 2\pi f$	$-\infty < \Omega < \infty$
f	Cycles per second (Hz)	$f = \frac{F f_s}{2} = \frac{\omega}{2\pi T}$	$-\infty < f < \infty$
ω	Radians per sample	$\omega = \Omega T = \pi F$	$-\pi \leq \omega \leq \pi$
F	Cycles per sample	$F = \frac{f}{(f_s/2)}$	$-1 \leq F \leq 1$

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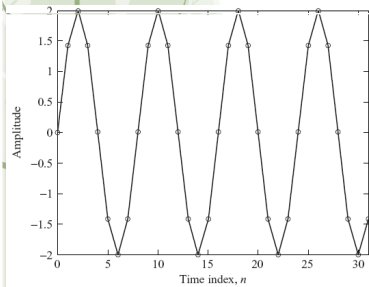
Contoh : Memperoleh 32 data sampling dari sinyal analog sinus

Bangkitkan 32 sample dari gelombang sinus dengan $A = 2$, $f = 1000$ Hz
 $f_s = 8$ kHz menggunakan Matlab

$$F = \frac{f}{(f_s/2)} = 0.25 \quad \omega = \pi F = 0.25\pi$$

Menggunakan MATLAB diperoleh:

$$x(n) = 2 \sin(\omega n), n = 0, 1, \dots, 31$$



```
n = [0:31]
omega = 0.25*pi
xn = 2*sin(omega*n)
plot(n, xn, 'o')

; % Time index n
; % Digital frequency
; % Sinewave generation
; % Samples are marked by
; % 'o'
```

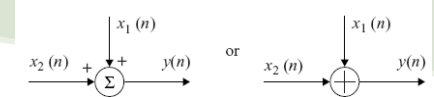
```
xlabel('Time index, n');
ylabel('Amplitude');
axis([0 31 -2 2]);
save sine.dat xn -ascii ;
```

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Operasi Matematik Sinyal Diskrit

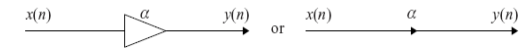
- Penjumlahan :**

$$y(n) = x_1(n) + x_2(n)$$



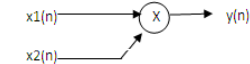
- Amplitudo scaling :**

$$y(n) = ax(n)$$



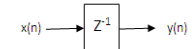
- Perkalian :**

$$y(n) = x_1(n) \cdot x_2(n)$$



- Delay negatif dan delay positif :**

$$y(n) = x(n-d); y(n) = x(n+d)$$



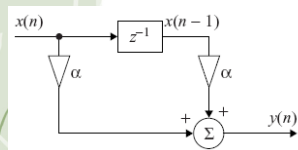
- Konvolusi diskrit :**

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

$$= \sum_{l=-\infty}^{\infty} x(l)h(n-l) = \sum_{l=-\infty}^{\infty} h(l)x(n-l)$$

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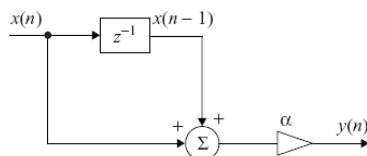
Contoh : Gambarkan arsitektur $y(n) = \alpha x(n) + \alpha x(n-1)$.



```
amov #alpha,XAR1 ; AR1 points to alpha
amov #temp,XAR2 ; AR2 points to temp
amov #yn,XAR4 ; AR4 points to yn
mov *(x1n),AC0 ; AC0 = x1(n)
add *(x2n),AC0 ; AC0 = x1(n)+x2(n)
mov AC0,*AR2 ; temp = x1(n)+x2(n),
; pointed by AR2

mpy *AR1,*AR2,AC1 ; AC1 = alpha*[x1(n)+x2(n)]
mov AC1,*AR4 ; yn = alpha*[x1(n)+x2(n)]
```

Lebih efektif jika : $y(n) = \alpha [x(n) + x(n-1)]$



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Sifat Sistem

- Linear Time Invariant:**
memiliki sifat superposisi dan tiap koefisien konstan
- Kasualitas:**
Output hanya bergantung dari input sekarang dan kondisi akan datang

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Konsep Sistem Linear Time Invariant

- $h(n)$ adalah respon impulse, output dari sistem jika inputnya impulse



Contoh tentukan koefisien respon impulse dari :

$$y(n) = b_0x(n) + b_1x(n-1) + b_2x(n-2).$$

Jawab:

$$h(0) = y(0) = b_0 \cdot 1 + b_1 \cdot 0 + b_2 \cdot 0 = b_0$$

$$h(1) = y(1) = b_0 \cdot 0 + b_1 \cdot 1 + b_2 \cdot 0 = b_1$$

$$h(2) = y(2) = b_0 \cdot 0 + b_1 \cdot 0 + b_2 \cdot 1 = b_2$$

$$h(3) = y(3) = b_0 \cdot 0 + b_1 \cdot 0 + b_2 \cdot 0 = 0$$

$\vdots$

$$h(n) = \{b_0, b_1, b_2, 0, 0, \dots\}.$$

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Filter

- FIR (Finite Impulse Response) / Recursive

$$y(n) = b_0x(n) + b_1x(n-1) + \dots + b_{L-1}x(n-L+1)$$

$$= \sum_{l=0}^{L-1} b_l x(n-l)$$

- IIR (Infinite Impulse Response) / Non-recursive

$$y(n) = b_0x(n) + b_1x(n-1) + \dots + b_{L-1}x(n-L+1) - a_1y(n-1) - \dots - a_My(n-M)$$

$$= \sum_{l=0}^{L-1} b_l x(n-l) - \sum_{m=1}^M a_m y(n-m).$$

Fungsi Matlab:

FIR:

```
yn = filter(b, 1, xn);
```

IIR:

```
yn = filter(b, a, xn);
```

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Contoh: Persamaan digital I/O dinyatakan dengan $y(n) = bx(n) - ay(n-1)$

- Jika $y(n) = 0$ for $n < 0$ dan $x(n) = \delta(n)$

Maka persamaan output IIRnya adalah

$$y(0) = bx(0) - ay(-1) = b$$

$$y(1) = bx(1) - ay(0) = -ay(0) = -ab$$

$$y(2) = bx(2) - ay(1) = -ay(1) = a^2b$$

$\vdots$

Secara umum:

$$y(n) = (-1)^n a^n b, \quad n = 0, 1, 2, \dots, \infty.$$

dengan a dan b tidak nol

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Transformasi Z

- Transformasi z untuk sinyal digital $x(n), -\infty < n < \infty$, dinyatakan dengan deret:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}.$$

- z adalah variabel kompleks yang dalam koordinat polar dapat dinyatakan dengan

$$z = re^{j\theta}$$

- Transformasi Balik-z:

$$x(n) = ZT^{-1}[X(z)] = \frac{1}{2\pi j} \oint_C X(z)z^{n-1}dz,$$

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Sifat-sifat transformasi z

- Linieritas (superposisi):

$$\begin{aligned} \text{ZT}[a_1 x_1(n) + a_2 x_2(n)] &= a_1 \text{ZT}[x_1(n)] + a_2 \text{ZT}[x_2(n)] \\ &= a_1 X_1(z) + a_2 X_2(z), \end{aligned}$$

where a_1 and a_2 are constants.

- Time-Shifting : $y(n) = x(n-k)$

$$Y(z) = \text{ZT}[x(n-k)] = z^{-k} X(z).$$

- Konvolusi :

$$X(z) = X_1(z)X_2(z).$$

- Derivasi :

$$\text{Z}[f(n-k)] = \sum_{n=-\infty}^{\infty} f(n-k)z^{-n} = \sum_{n=-\infty}^{\infty} f(n)z^{-(n+k)} = z^{-k} \sum_{n=-\infty}^{\infty} f(n)z^{-n} = z^{-k} F(z)$$

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Contoh Soal derivasi:

Tuliskan transformasi z dari persamaan berikut:

$$y(n) - 2y(n-1] = 0.5x(n-1)$$

Jawab:

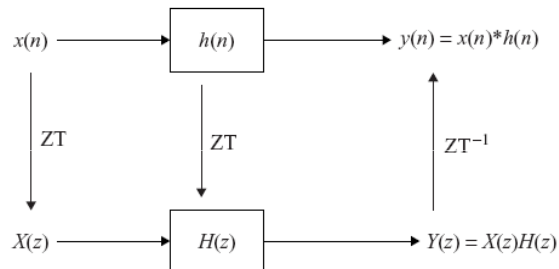
$$Y(z) = 2z^{-1}Y(z) + 0.5z^{-1}X(z)$$

Bisakah menuliskan fungsi transfernya?

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Fungsi Transfer pada Transformasi z

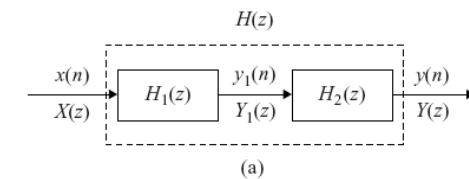
$$H(z) = \frac{Y(z)}{X(z)}.$$



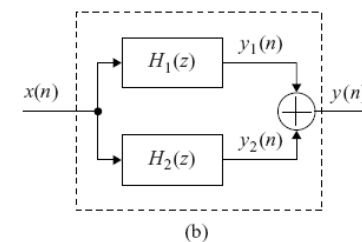
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Koneksi Cascade dan Paralel

$$H(z) = H_1(z)H_2(z) = H_2(z)H_1(z).$$



$$H(z) = H_1(z) + H_2(z).$$



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Table 4.1
Table of z-transforms of signals

Analog Signal	Sampled Signal	Z-transformed Signal
	$A\delta(n)$	A
$Au(t)$	$Au(n)$	$\frac{Az}{z-1}$
$Ae^{-at}u(t)$	$Ae^{-an}u(n)$	$\frac{Az}{z-e^{-aT}}$
	$Ae^{at}u(n)$	$\frac{Az}{z-e^a}$, $c = e^{-aT}$
$Atu(t)$	$AnTu(n)$	$\frac{ATz}{(z-1)^2}$
$A\cos(\omega t)u(t)$	$A\cos(\omega Tn)u(n)$	$\frac{Az[z-\cos(\omega T)]}{z^2-2z\cos(\omega T)+1}$
$A\sin(\omega t)u(t)$	$A\sin(\omega Tn)u(n)$	$\frac{Az\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$Ae^{-at}\cos(\omega t + \alpha)u(t)$	$Ae^{-an}\cos(\omega Tn + \alpha)$	$\frac{Az[z\cos(\alpha) - c\cos(\alpha - \omega T)]}{z^2-2cz\cos(\omega T)+c^2}$

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Contoh menggunakan Tabel Transformasi z

Input pada ADC dengan Perioda Sampling: $T=0,5$ detik adalah

$$x(t) = 7e^{-3t}u(t)$$

Tuliskan bentuk transformasi dari $X(z)$:

Jawab: untuk $T = 0,5$ maka $A = 7$ dan $a = 1,5$ sehingga

$$x(n) = 7e^{-1.5n}u(n)$$

$$X(z) = \frac{7z}{z - e^{-1.5}}$$

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Example 3.10: The LTI system with transfer function

$$H(z) = 1 - 2z^{-1} + z^{-3}$$

can be factored as

$$H(z) = (1 - z^{-1})(1 - z^{-1} - z^{-2}) = H_1(z)H_2(z).$$

Thus, the overall system $H(z)$ can be realized as the cascade of the first-order system $H_1(z) = 1 - z^{-1}$ and the second-order system $H_2(z) = 1 - z^{-1} - z^{-2}$.

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FIR dalam bentuk transformasi z

- Persamaan FIR :

$$y(n) = b_0x(n) + b_1x(n-1) + \dots + b_{L-1}x(n-L+1)$$

$$= \sum_{l=0}^{L-1} b_l x(n-l).$$

- Dapat bentuk transformasi z

$$Y(z) = b_0X(z) + b_1z^{-1}X(z) + \dots + b_{L-1}z^{-(L-1)}X(z)$$

$$= (b_0 + b_1z^{-1} + \dots + b_{L-1}z^{-(L-1)})X(z).$$

- Sehingga fungsi transfer untuk FIR:

$$H(z) = b_0 + b_1z^{-1} + \dots + b_{L-1}z^{-(L-1)} = \sum_{l=0}^{L-1} b_l z^{-l}.$$

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IIR dalam bentuk transformasi z

- Persamaan IIR

$$y(n) = b_0x(n) + b_1x(n-1) + \dots + b_{L-1}x(n-L+1) - a_1y(n-1) - \dots - a_My(n-M)$$

$$= \sum_{l=0}^{L-1} b_lx(n-l) - \sum_{m=1}^M a_my(n-m).$$

- Dalam bentuk transformasi z :

$$Y(z) = b_0X(z) + b_1z^{-1}X(z) + \dots + b_{L-1}z^{-L+1}X(z) - a_1z^{-1}Y(z) - \dots - a_Mz^{-M}Y(z)$$

$$= \left(\sum_{l=0}^{L-1} b_lz^{-l} \right) X(z) - \left(\sum_{m=1}^M a_mz^{-m} \right) Y(z).$$

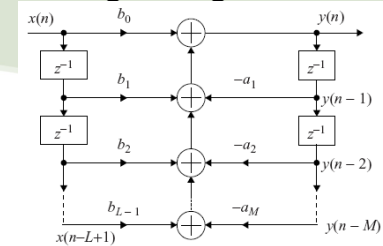
- Sehingga fungsi transfer untuk Filter IIR

$$H(z) = \frac{\sum_{l=0}^{L-1} b_lz^{-l}}{1 + \sum_{m=1}^M a_mz^{-m}} = \frac{\sum_{l=0}^{L-1} b_lz^{-l}}{\sum_{m=0}^M a_mz^{-m}},$$

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Contoh : Perhatikan Moving average filter

$$Y(z) = \frac{1}{L} \sum_{l=0}^{L-1} z^{-l} X(z).$$



Using the geometric series defined in Appendix A, the transfer function of the filter can be expressed as

$$H(z) = \frac{1}{L} \sum_{l=0}^{L-1} z^{-l} = \frac{1}{L} \left(\frac{1 - z^{-L}}{1 - z^{-1}} \right) = \frac{Y(z)}{X(z)}. \quad (3.43)$$

This equation can be rearranged as

$$Y(z) = z^{-1}Y(z) + \frac{1}{L} [X(z) - z^{-L}X(z)].$$

Taking the inverse z-transform of both sides, we obtain

$$y(n) = y(n-1) + \frac{1}{L} [x(n) - x(n-L)].$$

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Pole dan Zeros

- Persamaan Filter IIR dapat difaktorisasi menjadi

$$H(z) = b_0 \frac{\prod_{l=1}^{L-1} (z - z_l)}{\prod_{m=1}^M (z - p_m)} = \frac{b_0(z - z_1)(z - z_2) \dots (z - z_{L-1})}{(z - p_1)(z - p_2) \dots (z - p_M)}.$$

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Relation z-transform to Fourier transform

Fourier transform is the z-transform on the unit circle

$$X(e^{j\omega}) = X(z) \Big|_{z=e^{j\omega} \text{ or } |z|=1}$$

