**MODUL PERKULIAHAN**

**EDISI 1**

**LOGIKA MATEMATIKA**



Penulis :

Nelly Indriani Widiastuti S.Si., M.T.

JURUSAN TEKNIK INFORMATIKA

UNIVERSITAS KOMPUTER INDONESIA

BANDUNG

2011

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|  PETA KARNAUGH**10** |
| JUMLAH PERTEMUAN : 1 PERTEMUANTUJUAN INSTRUKSIONAL KHUSUS : |

## **PENDAHULUAN**

Metode Peta Karnaugh atau K-map merupakan metode grafis untuk menyederhanakan fungsi bolean. Metode ini ditemukan Maurice Karnaugh pada tahun 1953. Peta karnaugh adalah sebuah diagram yang terbentuk dari kotak-kotak tiap kotak merepresentasikan minterm. Tiap kotak dikatakan bertetangga jika minterm-minterm yang merepresentasikannya berbeda hanya 1 buah litaral.

Peta Karnaugh dapat dibentuk dari fungsi boolean yang dispesifikasikan dengan ekspresi boolean maupun fungsi yang dipresentasikan dengan tabel kebenaran.

## **Peta Karnaugh Dua Peubah**

Dua peubah dalam fungsi boolean adalah x dan y. Baris pada peta Karnaugh untuk peubah x dan kolom untuk y. Baris pertama diidentifikasi nilai 0 (menyatakan x’), sedangkan baris kedua dengan 1 (menyatakan x). Kolom pertama diidentifikasi 0 (menyetakan y’) sedangkan kolom kedua dengan 1 (menyatakan y).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *m*0 | *m*1 | *x*  0 | *x*’*y*’ | *x*’*y* |
|  | *m*2 | *m*3 | 1  | *xy*’ | *xy* |

## **Peta dengan tiga peubah**

Fungsi Boolean dengan tiga peubah memiliki jumlah kotak 23 = 8. Baris pada peta Karnaugh untuk peubah x dan kolom untuk peubah yz. Perhatikan urutan mi-nya, urutan disusun sedemikian rupa sehingga setiap dua kotak yang bertetangga hanya berbeda 1 bit.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  | *yz*00 | 01 | 11 | 10 |
|  | *m*0 | *m*1 | *m*3 | *m*2 |  | *x* 0  | *x*’*y*’*z*’ | *x*’*y*’*z* | *x*’*yz* | *x*’*yz*’ |
|  | *m*4 | *m*5 | *m*7 | *m*6 |  | 1  | *xy*’*z*’ | *xy*’*z* | *xyz* | *xyz*’ |

**Contoh.** Diberikan tabel kebenaran, gambarkan Peta Karnaugh.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | *y* | *Z* | *f*(*x*, *y*, *z*) |  |  |
| 0 | 0 | 0 | 0 |  |  |
| 0 | 0 | 1 | 0 |  |  |
| 0 | 1 | 0 | 1 |  |  |
| 0 | 1 | 1 | 0 |  |  |
| 1 | 0 | 0 | 0 |  |  |
| 1 | 0 | 1 | 0 |  |  |
| 1 | 1 | 0 | 1 |  |  |
| 1 | 1 | 1 | 1 |  |  |
|  | *yz*00 | 01 | 11 | 10 |
| *x* 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
|  |  |  |  |  |

## **Peta Karnaugh Empat Peubah**

Empat peubah dalam fungsi boolean adalah w, x, y, z. Jumlah kotak menjadi 16 buah. Perhatikan urutan mi-nya. Baris pada peta karnaugh untuk peubah wx dan kolom untuk peubah yz.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  | *yz*00 | 01 | 11 | 10 |
|  | *m*0 | *m*1 | *m*3 | *m*2 |  *wx* 00 | *w*’*x*’*y*’*z*’ | *w*’*x*’*y*’*z* | *w*’*x*’*yz* | *w*’*x*’*yz*’ |
|  | *m*4 | *m*5 | *m*7 | *m*6 |  | 01  | *w*’*xy*’*z*’ | *w*’*xy*’*z* | *w*’*xyz* | *w*’*xyz*’ |
|  | *m*12 | *m*13 | *m*15 | *m*14 |  | 11 | *wxy*’*z*’ | *wxy*’*z* | *wxyz* | *wxyz*’ |
|  | *m*8 | *m*9 | *m*11 | *m*10 |  | 10 | *wx*’*y*’*z*’ | *wx*’*y*’*z* | *wx*’*yz* | *wx*’*yz*’ |

**Contoh**. Diberikan tabel kebenaran, gambarkan Peta Karnaugh.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *w* | *x* | *y* | *z* | *f*(*w*, *x*, *y*, *z*) |  |
| 0 | 0 | 0 | 0 | 0 |  |  |
| 0 | 0 | 0 | 1 | 1 |  |  |
| 0 | 0 | 1 | 0 | 0 |  |  |
| 0 | 0 | 1 | 1 | 0 |  |  |
| 0 | 1 | 0 | 0 | 0 |  |  |
| 0 | 1 | 0 | 1 | 0 |  |  |
| 0 | 1 | 1 | 0 | 1 |  |  |
| 0 | 1 | 1 | 1 | 1 |  |  |
| 1 | 0 | 0 | 0 | 0 |  |  |
| 1 | 0 | 0 | 1 | 0 |  |  |
| 1 | 0 | 1 | 0 | 0 |  |  |
| 1 | 0 | 1 | 1 | 0 |  |  |
| 1 | 1 | 0 | 0 | 0 |  |  |
| 1 | 1 | 0 | 1 | 0 |  |  |
| 1 | 1 | 1 | 0 | 1 |  |  |
| 1 | 1 | 1 | 1 | 0 |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 1 | 0 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 |

## **Teknik minimisasi fungsi boolean dengan peta karnaugh**

1. *Pasangan*: dua buah 1 yang bertetangga

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | **1**1 | **1**0 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| **11** | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

*Sebelum disederhanakan*: *f*(*w*, *x*, *y*, *z*) = *wxyz* + *wxyz*’

*Hasil Penyederhanaan*: *f*(*w*, *x*, *y*, *z*) = *wxy*

Bukti secara aljabar:

 *f*(*w*, *x*, *y*, *z*) = *wxyz* + *wxyz*’

 = *wxy*(*z* + *z*’)

 = *wxy*(1)

 = *wxy*

2. *Kuad*: empat buah 1 yang bertetangga

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| **11** | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

*Sebelum disederhanakan*: *f*(*w*, *x*, *y*, *z*) = *wxy*’*z*’ + *wxy*’*z* + *wxyz* + *wxyz*’

*Hasil penyederhanaan*: *f*(*w*, *x*, *y*, *z*) = *wx*

Bukti secara aljabar:

 *f*(*w*, *x*, *y*, *z*) = *wxy*’ + *wxy*

 = *wx*(*z*’ + *z*)

 = *wx*(1)

 = *wx*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx*  00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

Contoh lain:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz***0**0 | **0**1 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| **1**1 | 1 | 1 | 0 | 0 |
| **1**0 | 1 | 1 | 0 | 0 |

Sebelum disederhanakan: *f*(*w*, *x*, *y*, *z*) = *wxy*’*z*’ + *wxy*’*z* + *wx*’*y*’*z*’ + *wx*’*y*’z

*Hasil penyederhanaan*: *f*(*w*, *x*, *y*, *z*) = *wy*’

3. *Oktet*: delapan buah1 yang bertetangga

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| **1**1 | 1 | 1 | 1 | 1 |
| **1**0 | 1 | 1 | 1 | 1 |

*Sebelum disederhanakan*: *f*(*a*, *b*, *c*, *d*) = *wxy*’*z*’ + *wxy*’*z* + *wxyz* + *wxyz*’+*wx*’*y*’*z*’ + *wx*’*y*’*z* + *wx*’*yz* + *wx*’*yz*’

*Hasil penyederhanaan*: *f*(*w*, *x*, *y*, *z*) = *w*

Bukti secara aljabar:

 *f*(*w*, *x*, *y*, *z*) = *wy*’ + *wy*

 = *w*(*y*’ + *y*)

 = *w*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx*  00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 |

**Contoh 5.11.** Sederhanakan fungsi Boolean *f*(*x*, *y*, *z*) = *x*’*yz* + *xy*’*z*’ + *xyz* + *xyz*’.

Jawab:

 Peta Karnaugh untuk fungsi tersebut adalah:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *x* 0 |  |  | 1 |  |
| 1 | 1 |  | 1 | 1 |

Hasil penyederhanaan: *f*(*x*, *y*, *z*) = *yz* + *xz*’

**Contoh 5.12.** Andaikan suatu tabel kebenaran telah diterjemahkan ke dalam Peta Karnaugh. Sederhanakan fungsi Boolean yang bersesuaian sesederhana mungkin.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 1 | 1 | 1 |
| 01 | 0 | 0 | 0 | 1 |
| 11 | 1 | 1 | 0 | 1 |
| 10 | 1 | 1 | 0 | 1 |

Jawab: (lihat Peta Karnaugh) *f*(*w*, *x*, *y*, *z*) = *wy*’ + *yz*’ + *w*’*x*’*z*

**Contoh 5.13.** Minimisasi fungsi Boolean yang bersesuaian dengan Peta Karnaugh di bawah ini.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 |

Jawab: (lihat Peta Karnaugh) *f*(*w*, *x*, *y*, *z*) = *w* + *xy*’*z*

Jika penyelesaian Contoh 5.13 adalah seperti di bawah ini:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx*  00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 |

maka fungsi Boolean hasil penyederhanaan adalah

 *f*(*w*, *x*, *y*, *z*) = *w* + *w*’*xy*’*z* (jumlah literal = 5)

yang ternyata masih belum sederhana dibandingkan *f*(*w*, *x*, *y*, *z*) = *w* + *xy*’*z* (jumlah literal = 4).

**Contoh 5.14.** (Penggulungan/*rolling*) Sederhanakan fungsi Boolean yang bersesuaian dengan Peta Karnaugh di bawah ini.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 |

Jawab: *f*(*w*, *x*, *y*, *z*) = *xy*’*z*’ + *xyz*’ ==> belum sederhana

Penyelesaian yang lebih minimal:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*0**0** | 01 | 11 | 1**0** |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 0**1** | 1 | 0 | 0 | 1 |
| 1**1** | 1 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 |

 *f*(*w*, *x*, *y*, *z*) = *xz*’ ===> lebih sederhana

**Contoh 5.15**: (Kelompok berlebihan) Sederhanakan fungsi Boolean yang bersesuaian dengan Peta Karnaugh di bawah ini.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx* 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 |

Jawab: *f*(*w*, *x*, *y*, *z*) = *xy*’*z* + *wxz* + *wyz* → masih belum sederhana.

Penyelesaian yang lebih minimal:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *wx*  00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 |

 *f*(*w*, *x*, *y*, *z*) = *xy*’*z* + *wyz* ===> lebih sederhana

**Contoh 5.16.** Sederhanakan fungsi Boolean yang bersesuaian dengan Peta Karnaugh di bawah ini.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *cd*00 | 01 | 11 | 10 |
| *ab*  00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 1 | 1 | 1 |

Jawab: (lihat Peta Karnaugh di atas) *f*(*a*, *b*, *c*, *d*) = *ab* + *ad* + *ac* + *bcd*

**Contoh 5.17.** Minimisasi fungsi Boolean *f*(*x*, *y*, *z*) = *x*’*z* + *x*’*y* + *xy*’*z* + *yz*

Jawab:

*x’z* = *x*’*z*(*y* + *y*’) = *x*’*yz* + *x*’*y*’*z*

*x*’*y* = *x*’*y*(*z* + *z*’) = *x*’*yz* + *x*’*yz*’

*yz* = *yz*(*x* + *x*’) = *xyz* + *x*’*yz*

*f*(*x*, *y*, *z*) = *x*’*z* + *x*’*y* + *xy*’*z* + *yz*

 = *x*’*yz* + *x*’*y*’*z* + *x*’*yz* + *x*’*yz*’ + *xy*’*z* + *xyz* + *x*’*yz*

 = *x*’*yz* + *x*’*y*’*z* + *x*’*yz*’ + *xyz* + *xy*’*z*

 Peta Karnaugh untuk fungsi tersebut adalah:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *yz*00 | 01 | 11 | 10 |
| *x* 0 |  | 1 | 1 | 1 |
| 1 |  | 1 | 1 |  |

Hasil penyederhanaan: *f*(*x*, *y*, *z*) = *z* + *x*’*yz*’

# Peta Karnaugh untuk lima peubah

 000 001 011 010 110 111 101 100

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 00 | *m*0 | *m*1 | *m*3 | *m*2 | *m*6 | *m*7 | *m*5 | *m*4 |
| 01 | *m*8 | *m*9 | *m*11 | *m*10 | *m*14 | *m*15 | *m*13 | *m*12 |
| 11 | *m*24 | *m*25 | *m*27 | *m*26 | *m*30 | *m*31 | *m*29 | *m*28 |
| 10 | *m*16 | *m*17 | *m*19 | *m*18 | *m*22 | *m*23 | *m*21 | *m*20 |
|  |  |  |  |  |  |  |  |  |

 Garis pencerminan

**Contoh 5.21.** (Contoh penggunaan Peta 5 peubah) Carilah fungsi sederhana dari *f*(*v*, *w*, *x*, *y*, *z*) = Σ (0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)

Jawab:

 Peta Karnaugh dari fungsi tersebut adalah:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | *xyz*000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 |  |
|  | *vw* 00 | 1 |  |  | 1 | 1 |  |  | 1 |  |
|  | 01 |  | 1 | 1 |  |  | 1 | 1 |  |   |
|  | 11 |  | 1 | 1 |  |  | 1 | 1 |  |  |
|  | 10 |  | 1 |  |  |  |  | 1 |  |  |

 Jadi *f*(*v*, *w*, *x*, *y*, *z*) = *wz* + *v*’*w*’*z*’ + *vy*’*z*