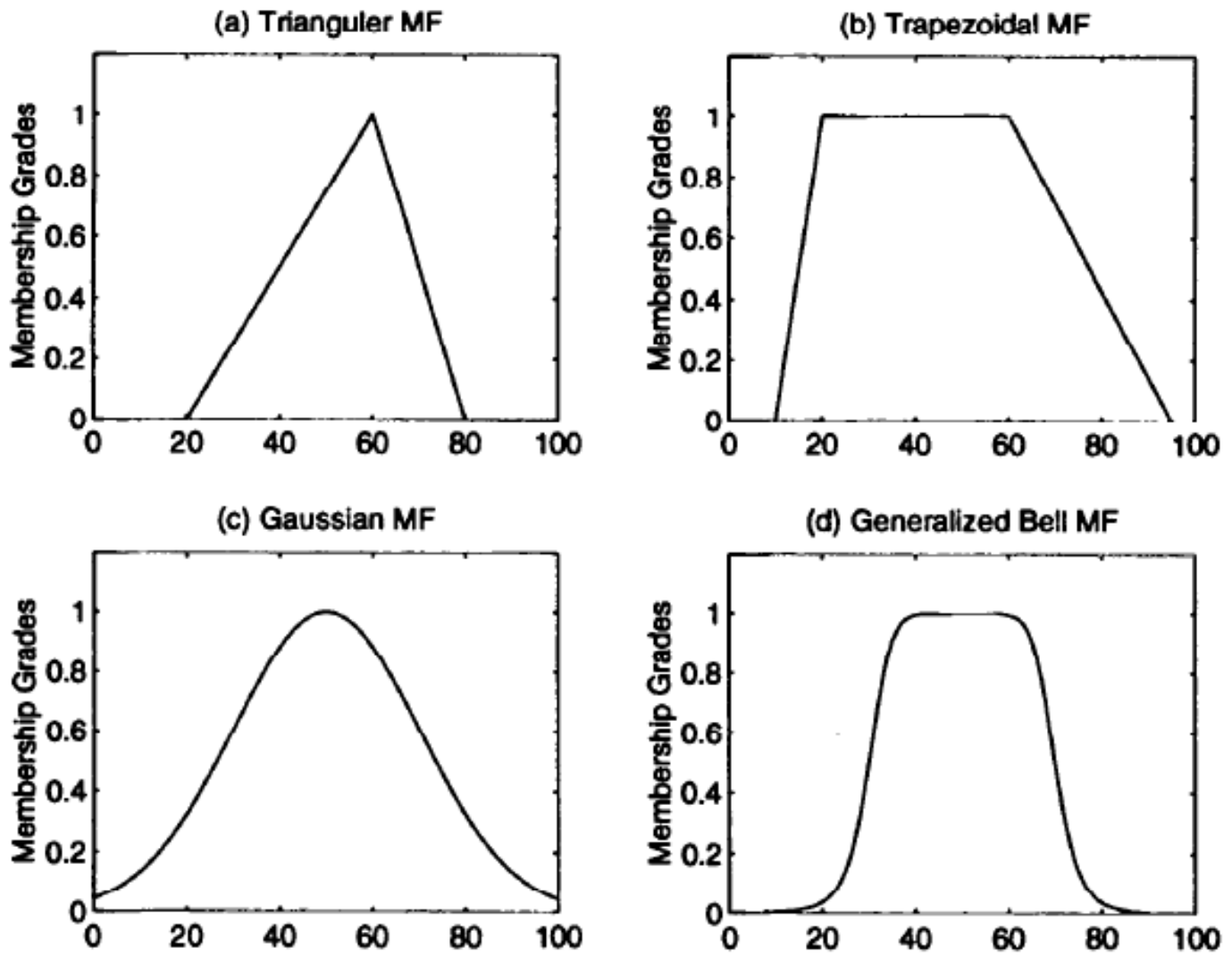
SISTEM FUZZY

Fungsi Keanggotaan Fuzzy



(a) $triangle(x; 20, 60, 80)$;

(b) $trapezoid(x; 10, 20, 60, 95)$;

(c) $gaussian(x; 50, 20)$;

(d) $bell(x; 20, 4, 50)$

A **triangular MF** is specified by three parameters $\{a, b, c\}$ as follows:

$$\text{triangle}(x; a, b, c) = \begin{cases} 0, & x \leq a. \\ \frac{x-a}{b-a}, & a \leq x \leq b. \\ \frac{c-x}{c-b}, & b \leq x \leq c. \\ 0, & c \leq x. \end{cases} \tag{2.18}$$

By using min and max, we have an alternative expression for the preceding equation:

$$\text{triangle}(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right) \tag{2.19}$$

The parameters $\{a, b, c\}$ (with $a < b < c$) determine the x coordinates of the three corners of the underlying triangular MF.

A **trapezoidal MF** is specified by four parameters $\{a, b, c, d\}$ as follows:

$$\text{trapezoid}(x; a, b, c, d) = \begin{cases} 0, & x \leq a. \\ \frac{x-a}{b-a}, & a \leq x \leq b. \\ 1, & b \leq x \leq c. \\ \frac{d-x}{d-c}, & c \leq x \leq d. \\ 0, & d \leq x. \end{cases} \tag{2.20}$$

An alternative concise expression using min and max is

$$\text{trapezoid}(x; a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right). \tag{2.21}$$

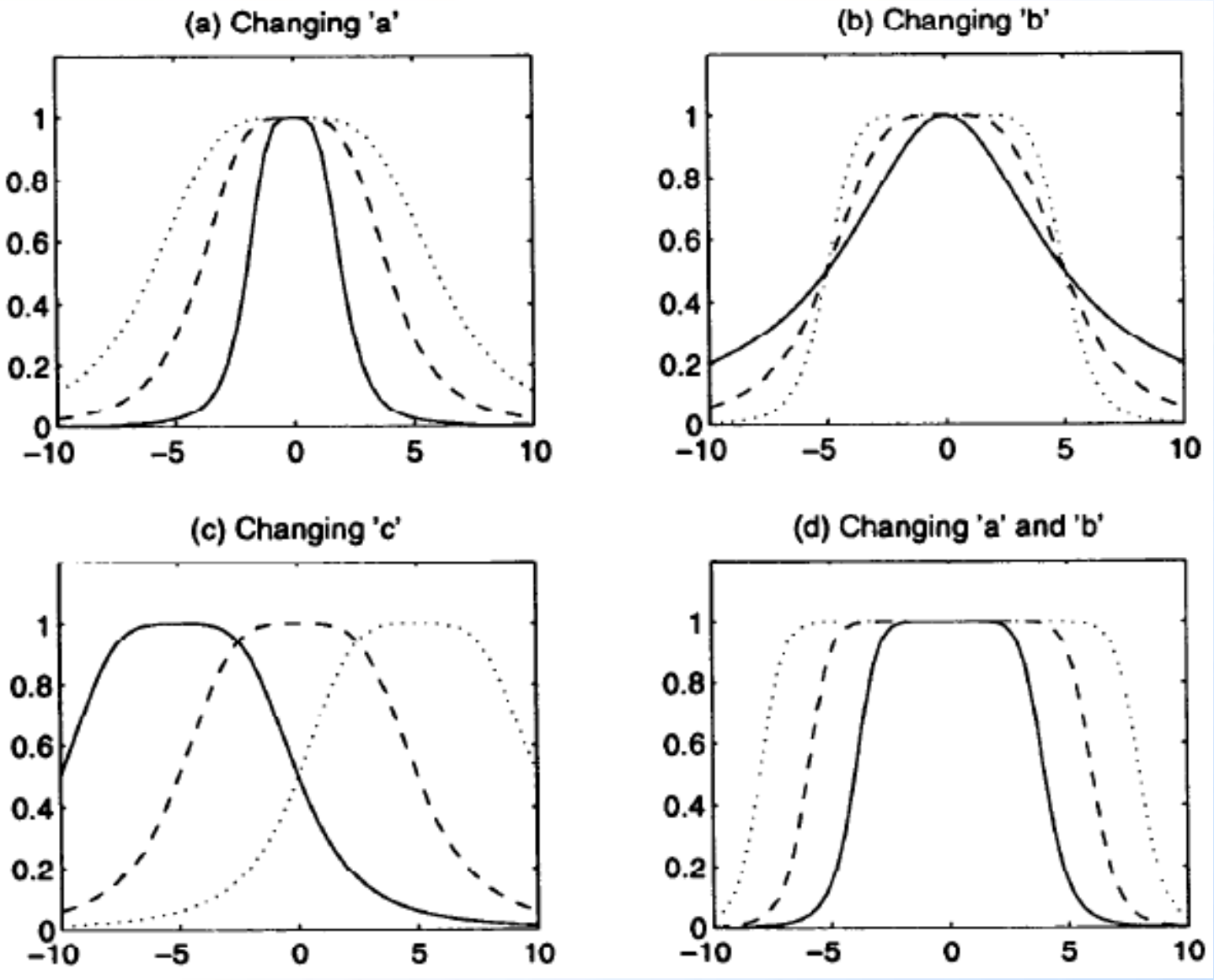
The parameters $\{a, b, c, d\}$ (with $a < b \leq c < d$) determine the x coordinates of the four corners of the underlying trapezoidal MF.

A **Gaussian MF** is specified by two parameters $\{c, \sigma\}$:

$$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$

A **generalized bell MF** (or **bell MF**) is specified by three parameters $\{a, b, c\}$:

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}},$$


(a) Changing 'a'
(b) Changing 'b'
(c) Changing 'c'
(d) Changing 'a' and 'b'
1
0.8
0.6
0.4
0.2
0
−10
−5
0
5
10

OPERASI LOGIKA PADA FUZZY

- Operator “AND”
- Operator “OR”

Operator "AND"

Four of the most frequently used T-norm operators are

Minimum: $T_{min}(a, b) = \min(a, b) = a \wedge b.$

Algebraic product: $T_{ap}(a, b) = ab.$

Operator “OR”

Maximum: $S(a,b) = \max(a,b) = a \vee b.$

Algebraic sum: $S(a,b) = a + b - ab.$

Contoh Inferensi Fuzzy

- Contoh 1 : (1 aturan, 1 syarat)
- Contoh 2 : (1 aturan, 2 syarat)
- Contoh 3 : (4 aturan @ 2 syarat)

Contoh 1 (1 Rules, 1 syarat)

- Kasus pengereman otomatis
- Fuzzy Rules :

  Jika **Jarak Dekat** maka **Rem Penuh**
- Didefinisikan :
  - MF Jarak Dekat : trapezodial (-2, -1, 4, 12)
  - Rem Penuh : 100 %
- Berapakah besar pengereman jika jarak = 6m ?

Jawab Contoh 1 :

- Mencari derajat keanggotaan dekat :

$$\mu_{dekat} = (6; -2, -1, 4, 12) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{6-(-2)}{(-1)-(-2)}, 1, \frac{12-6}{12-4}\right), 0\right) = \frac{3}{4}$$

- Tingkat pengereman :

$$Rem = \frac{3}{4} \times 100\% = 75\%$$

Contoh 2 : (1 Rules, 2 syarat)

- Kasus pengereman otomatis
- Fuzzy Rules :

  Jika **Kecepatan Tinggi** dan **Jarak Dekat** maka **Rem Penuh**
- Didefinisikan :
  - MF Jarak Dekat : trapezodial (-2, -1, 4, 12)
  - MF Kecepatan Tinggi : trapezoidal (40, 80, 110, 120)
  - Operator “DAN” yang digunakan : operator MINIMUM
  - Rem Penuh : 100 %
- Berapakah besar pengereman jika jarak = 6m dan kecepatan 60 km/jam ?

Jawab Contoh 2 :

- Mencari derajat keanggotaan jarak dekat :

$$\mu_{dekat} = (6; -2, -1, 4, 12) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{6-(-2)}{(-1)-(-2)}, 1, \frac{12-6}{12-4}\right), 0\right) = \frac{3}{4}$$

- Mencari derajat keanggotaan kecepatan tinggi :

$$\mu_{tinggi} = (60; 40, 80, 110, 120) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{60-40}{80-40}, 1, \frac{120-60}{120-110}\right), 0\right) = 0{,}5$$

- Tingkat pengereman :

$$Rem = min(\mu_{tinggi}, \mu_{dekat}) \times 100\% = min\left(\frac{3}{4}, \frac{1}{2}\right) \times 100\% = 50\%$$

Contoh 3 : (4 Rules, 2 syarat)

- Fuzzy Rules :
  - Jika Kecepatan Tinggi dan Jarak Dekat maka Rem Penuh
  - Jika Kecepatan Tinggi dan Jarak Jauh maka Rem Sedang
  - Jika Kecepatan Rendah dan Jarak Dekat maka Rem Sedang
  - Jika Kecepatan Rendah dan Jarak Jauh maka Rem Sedikit

FIS Sugeno

- MF Kecepatan Tinggi :

  trapezoidal (40, 80, 110, 120)
- MF Kecepatan Rendah :

  trapezoidal(-2, -1, 20, 60)
- MF Jarak Jauh :

  trapezoidal (8, 16, 21, 22)
- MF Jarak Dekat :

  trapezodial (-2, -1, 4, 12)
- MF Rem Penuh : 100 %
- MF Rem Sedang : 50 %
- MF Rem Sedikit : 20 %

Calculation Example

Kecepatan = 60

Jarak = 10

$$\mu_{tinggi} = (60; 40,80,110,120) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{60-40}{80-40}, 1, \frac{120-60}{120-110}\right), 0\right) = 0{,}5$$

$$\mu_{rendah} = (60; -2, -1,20,60) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{60-(-2)}{(-1)-(-2)}, 1, \frac{60-60}{60-20}\right), 0\right) = 0$$

$$\mu_{jauh} = (10; 8,16,21,22) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{10-8}{16-8}, 1, \frac{22-10}{22-21}\right), 0\right) = 0{,}25$$

$$\mu_{dekat} = (10; -2, -1,4,12) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{10-(-2)}{(-1)-(-2)}, 1, \frac{12-10}{12-4}\right), 0\right) = 0{,}25$$

$$w_1 = min\left(\mu_{tinggi}(60; 40,80,110,120), \mu_{dekat}(10; -2, -1,4,12)\right) = min(0.5, 0.25) = 0.25$$

$$w_2 = min\left(\mu_{tinggi}(60; 40,80,110,120), \mu_{jauh}(10; 8,16,21,22)\right) = min(0.5, 0.25) = 0.25$$

$$w_3 = min\left(\mu_{rendah}(60; -2, -1,20,60), \mu_{dekat}(10; -2, -1,4,12)\right) = min(0, 0.25) = 0$$

$$w_4 = min\left(\mu_{rendah}(60; -2, -1,20,60), \mu_{jauh}(10; 8,16,21,22)\right) = min(0, 0.25) = 0$$

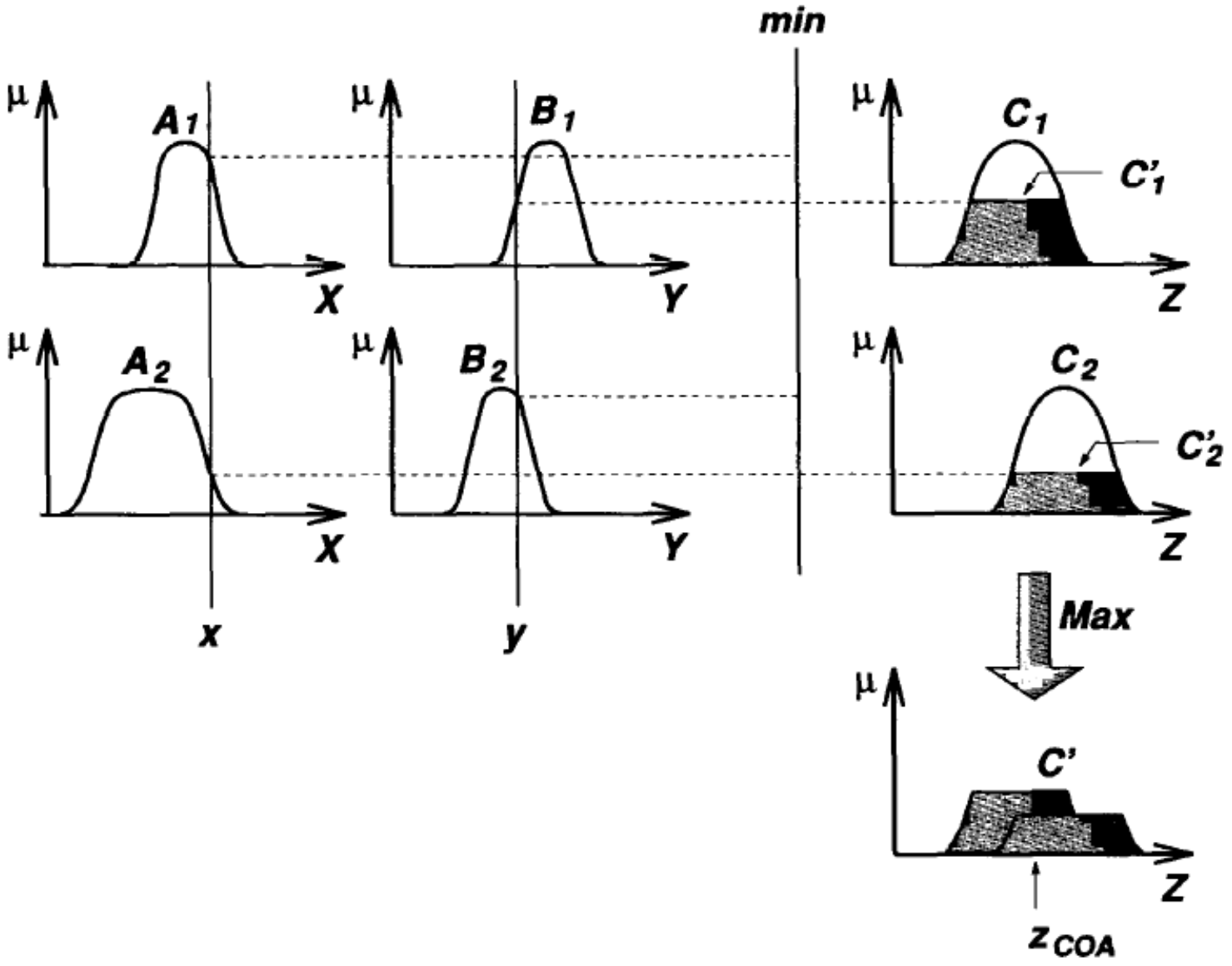
Defuzzification Sugeno :

$$out = \frac{0.25 \times 100 + 0.25 \times 50 + 0 \times 50 + 0 \times 20}{0.25 + 0.25 + 0 + 0} = \frac{37.5}{0.5} = 75$$

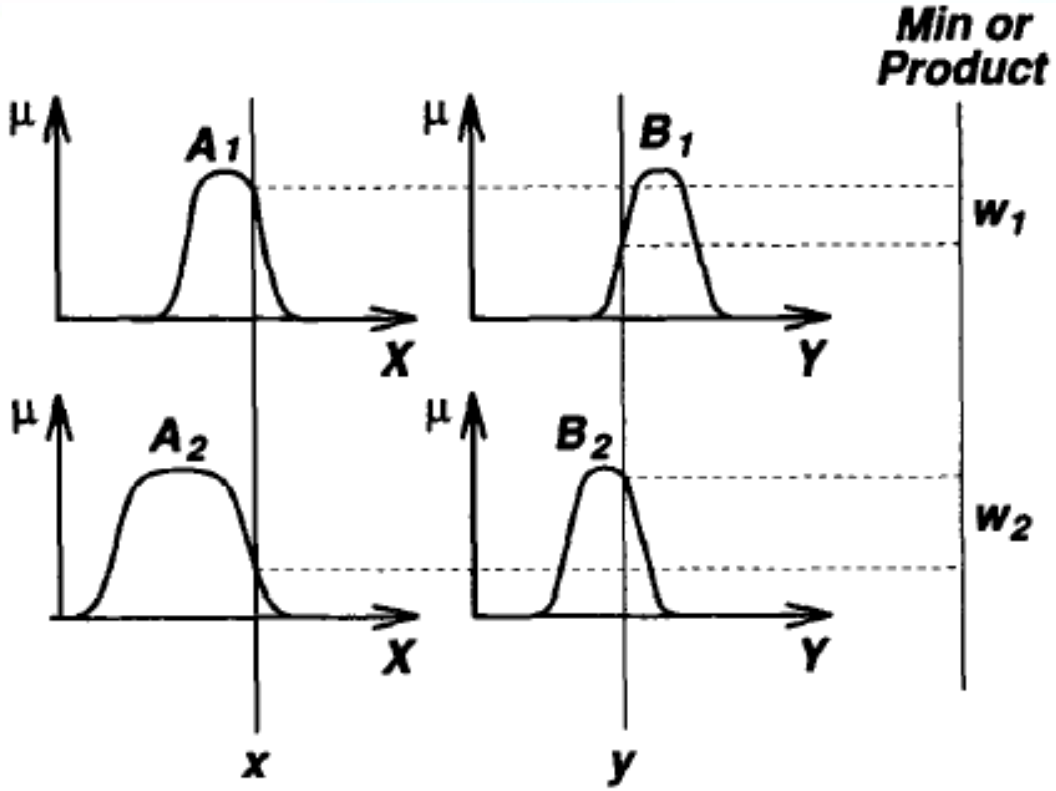
Fuzzy Inference System

- Steps in FIS :
  - Fuzzification
  - Fuzzy Logic Inference
  - Defuzzification

FIS Mamdani



FIS Sugeno



$$z_1 = p_1 x + q_1 y + r_1$$

$$z_2 = p_2 x + q_2 y + r_2$$

Weighted Average

$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

Proses pengereman otomatis

- Fuzzy Rules :
  - Jika Kecepatan Tinggi dan Jarak Dekat maka Rem Penuh
  - Jika Kecepatan Tinggi dan Jarak Jauh maka Rem Sedang
  - Jika Kecepatan Rendah dan Jarak Dekat maka Rem Sedang
  - Jika Kecepatan Rendah dan Jarak Jauh maka Rem Sedikit

FIS Sugeno

- MF Kecepatan Tinggi :

  trapezoidal (40, 80, 110, 120)
- MF Kecepatan Rendah :

  trapezoidal(-2, -1, 20, 60)
- MF Jarak Jauh :

  trapezoidal (8, 16, 21, 22)
- MF Jarak Dekat :

  trapezodial (-2, -1, 4, 12)
- MF Rem Penuh : 100 %
- MF Rem Sedang : 50 %
- MF Rem Sedikit : 20 %

FIS Mamdani

- MF Kecepatan Rendah :

  trapezoidal (-2, -1, 20, 60)
- MF Kecepatan Tinggi :

  trapezoidal (40, 80, 110, 120)
- MF Jarak Jauh :

  trapezoidal (8, 16, 21, 22)
- MF Jarak Dekat :

  trapezodial (-2, -1, 4, 12)
- MF Rem Penuh : triangle (80,90,100)
- MF Rem Sedang : triangle (40,50,60)
- MF Rem Sedikit : triangle (10,20,30)

Calculation Example

Kecepatan = 60

Jarak = 10

$$\mu_{tinggi} = (60; 40,80,110,120) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{60-40}{80-40}, 1, \frac{120-60}{120-110}\right), 0\right) = 0{,}5$$

$$\mu_{rendah} = (60; -2, -1,20,60) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{60-(-2)}{(-1)-(-2)}, 1, \frac{60-60}{60-20}\right), 0\right) = 0$$

$$\mu_{jauh} = (10; 8,16,21,22) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{10-8}{16-8}, 1, \frac{22-10}{22-21}\right), 0\right) = 0{,}25$$

$$\mu_{dekat} = (10; -2, -1,4,12) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) = max\left(min\left(\frac{10-(-2)}{(-1)-(-2)}, 1, \frac{12-10}{12-4}\right), 0\right) = 0{,}25$$

$$w_1 = min\left(\mu_{tinggi}(60; 40,80,110,120), \mu_{dekat}(10; -2, -1,4,12)\right) = min(0.5, 0.25) = 0.25$$

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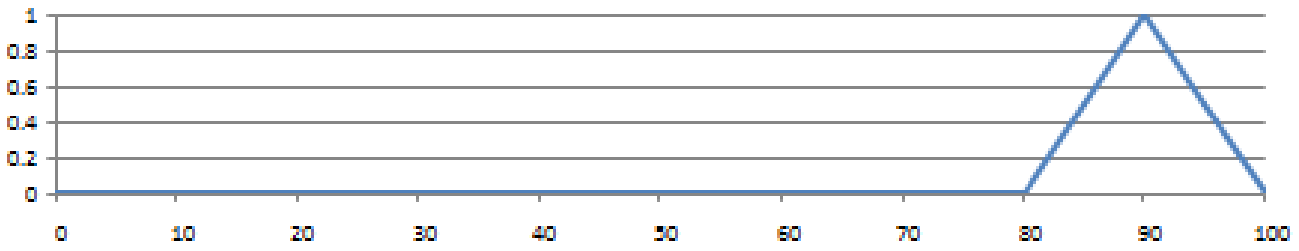
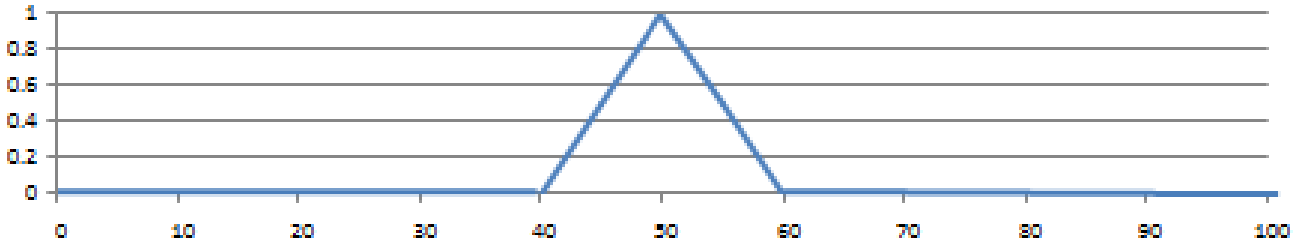
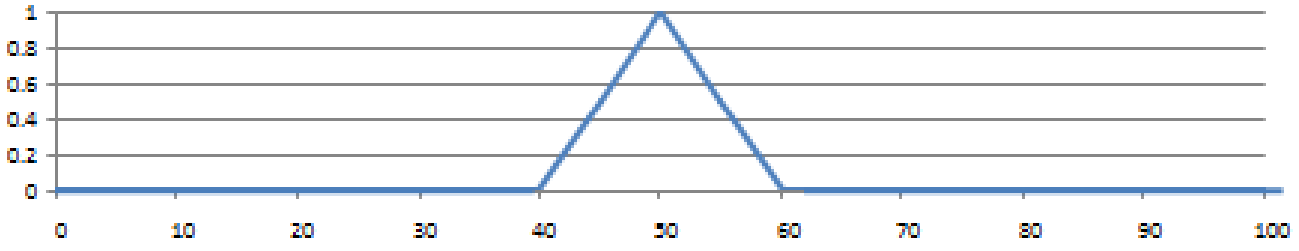


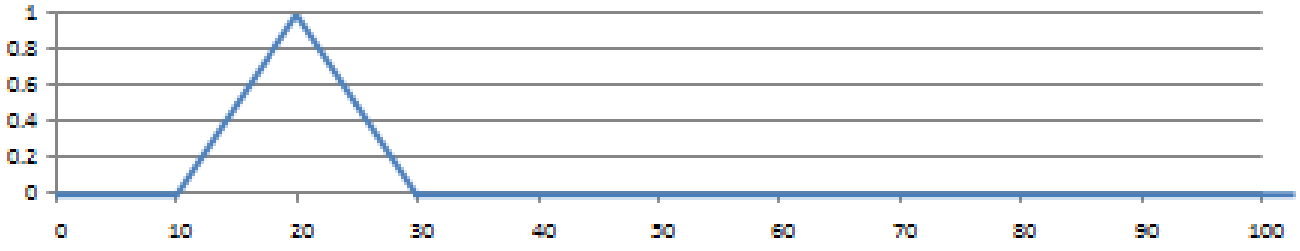
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	1	0.5	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



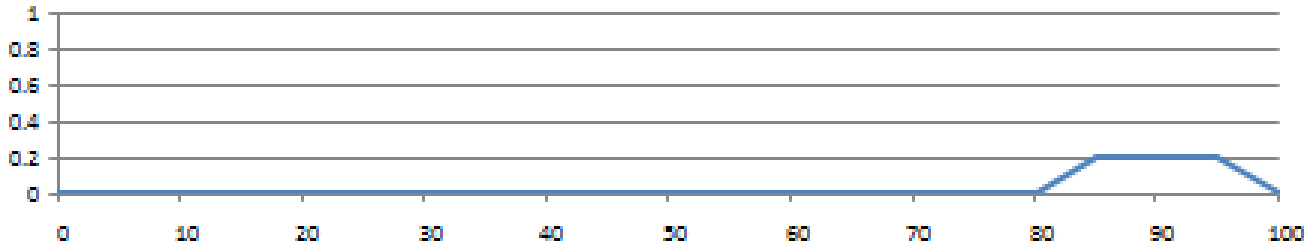
0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



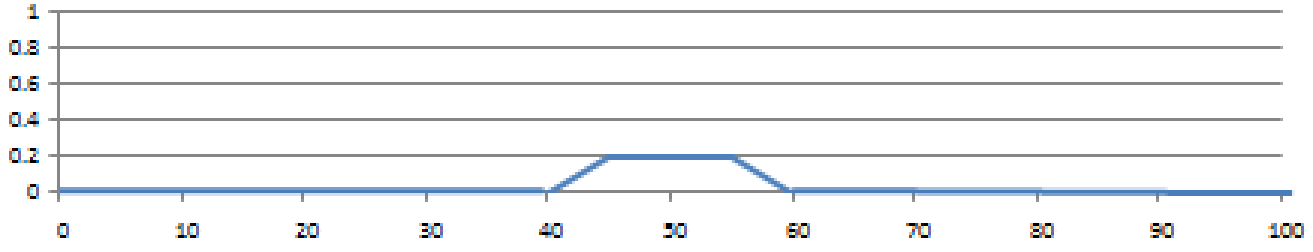
0	0	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



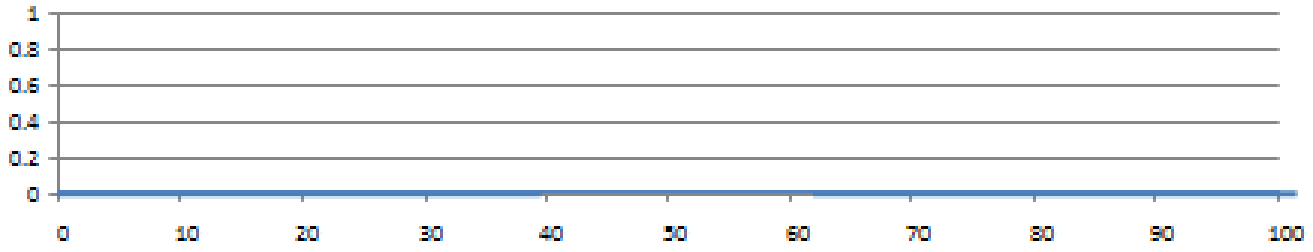
0	0	0	0.5	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



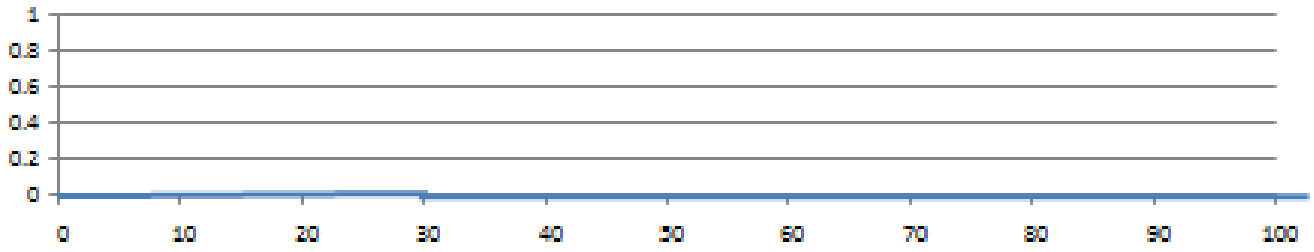
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0.2	0.2	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



0	0	0	0	0	0	0	0	0	0.2	0.2	0.2	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

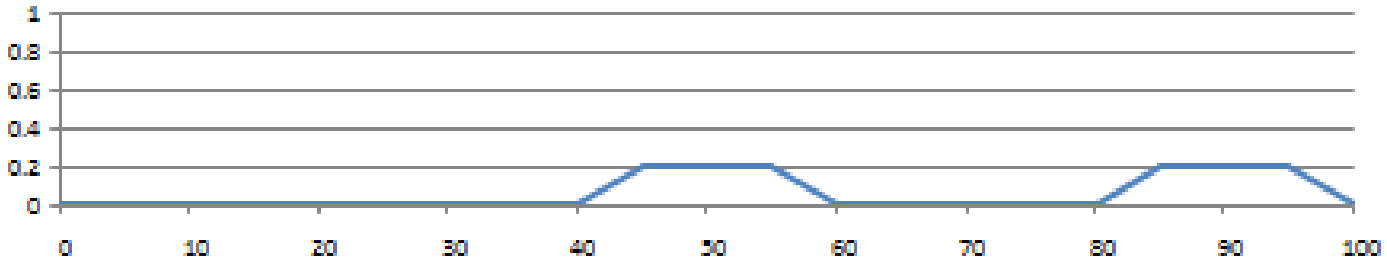


0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100



0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

Hasil Agregasi

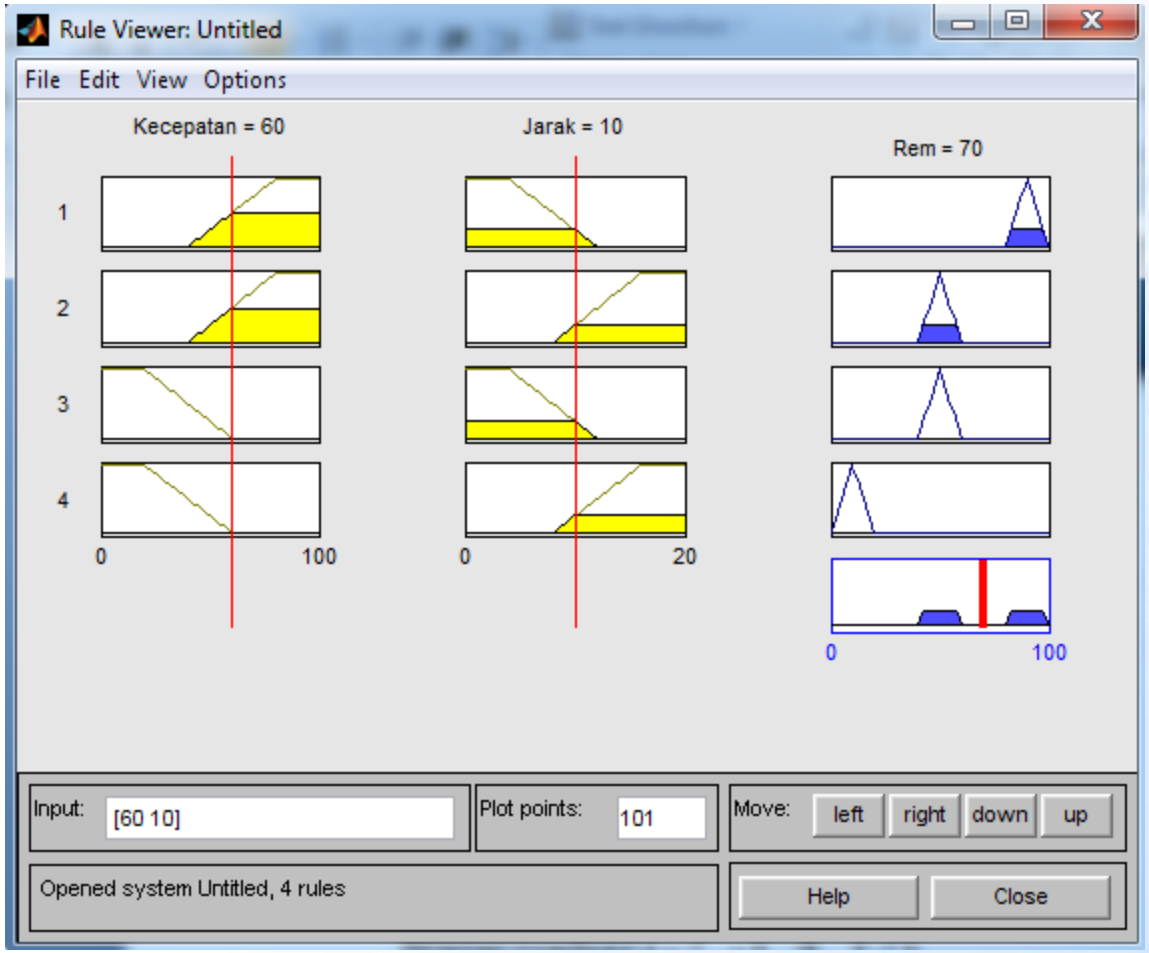


0	0	0	0	0	0	0	0	0	0.2	0.2	0.2	0	0	0	0	0	0.2	0.2	0.2	0
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

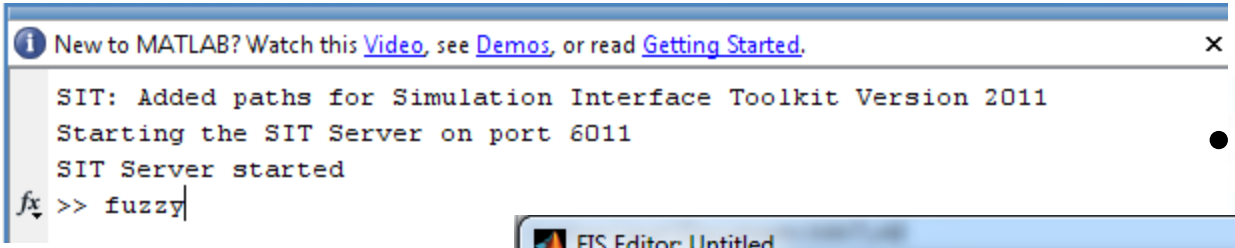
- Centroid Area :

$$out = \frac{0 \times 0 + 5 \times 0 + 10 \times 0 + \ldots + 90 \times 0.25 + 95 \times 0.25 + 100 \times 0}{0 + 0 + 0 + \ldots + 0.25 + 0.25 + 0} = \frac{84}{1.2} = 70$$

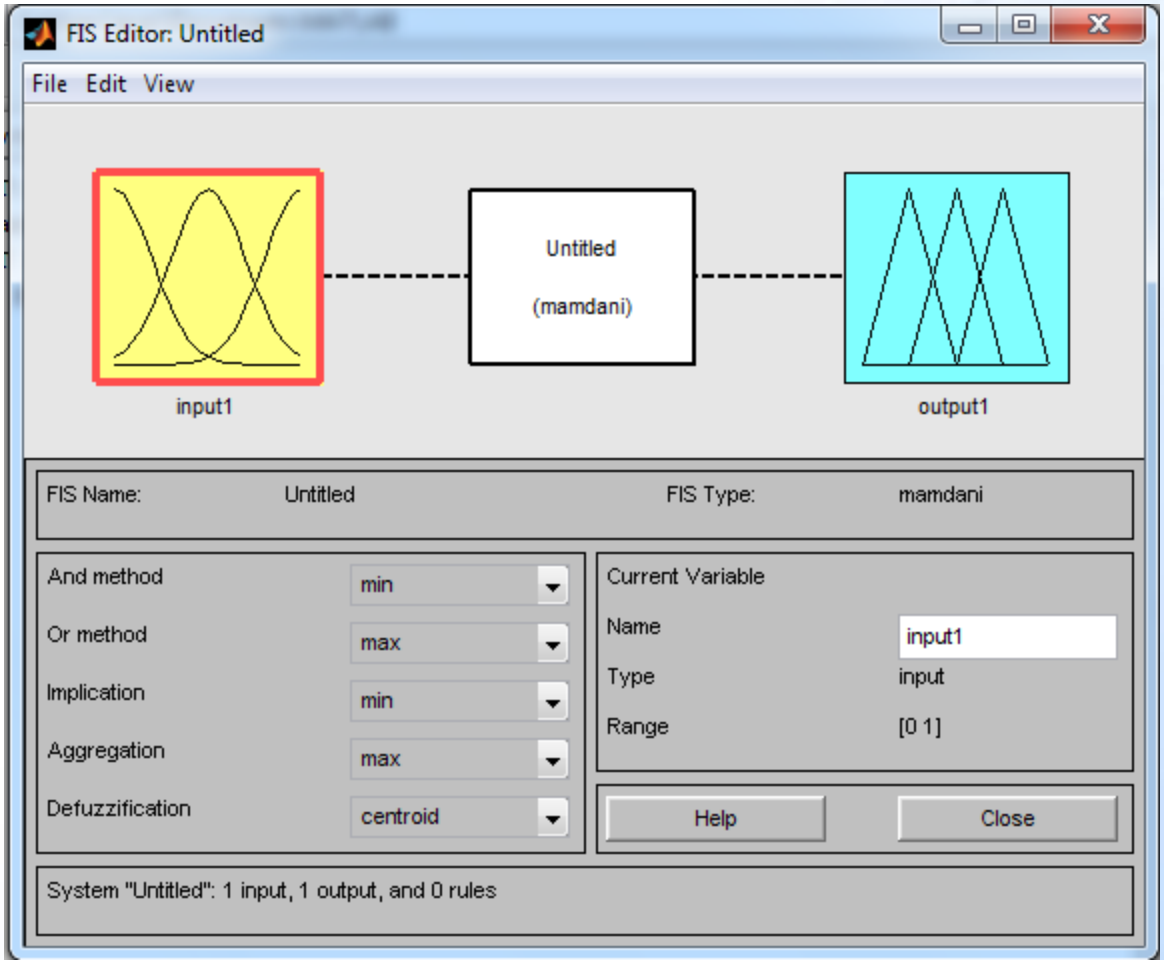
Perbandingan dgn simulasi MATLAB

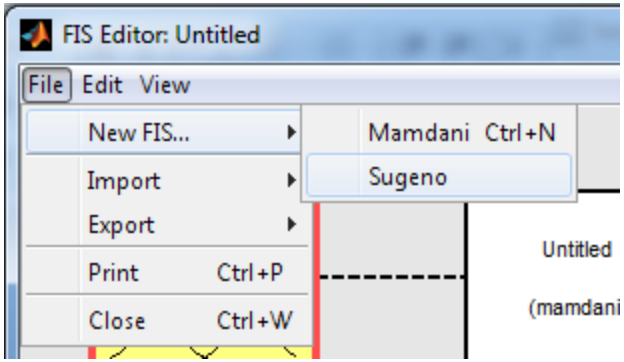


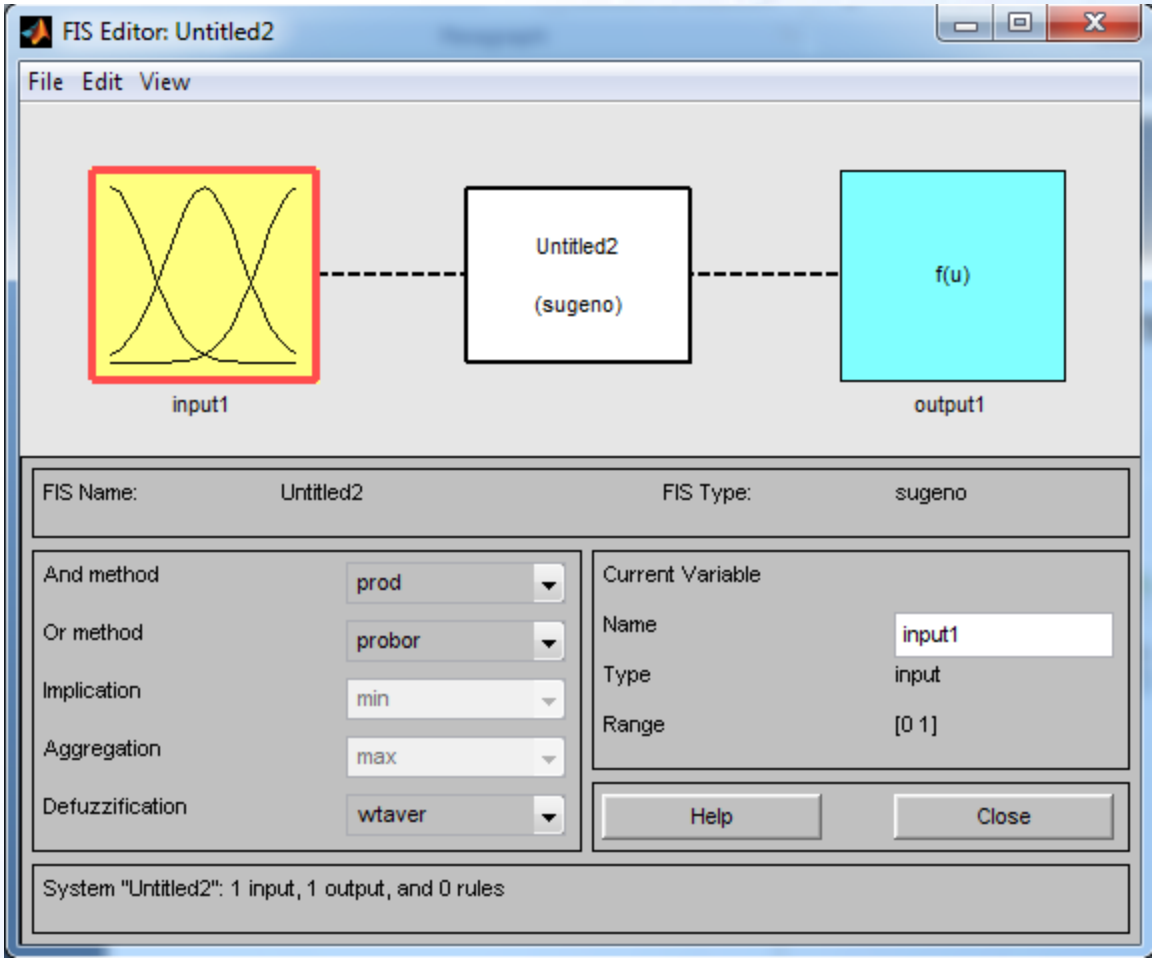
Toolbox Fuzzy pada MATLAB

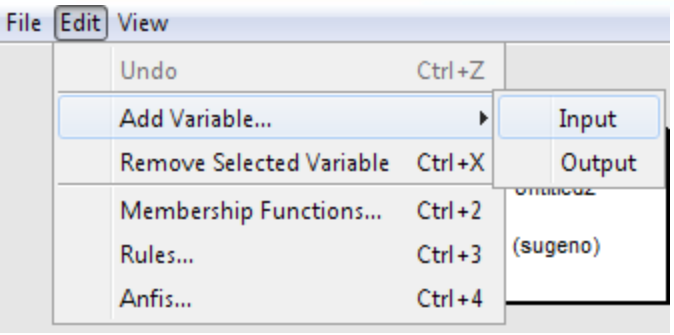


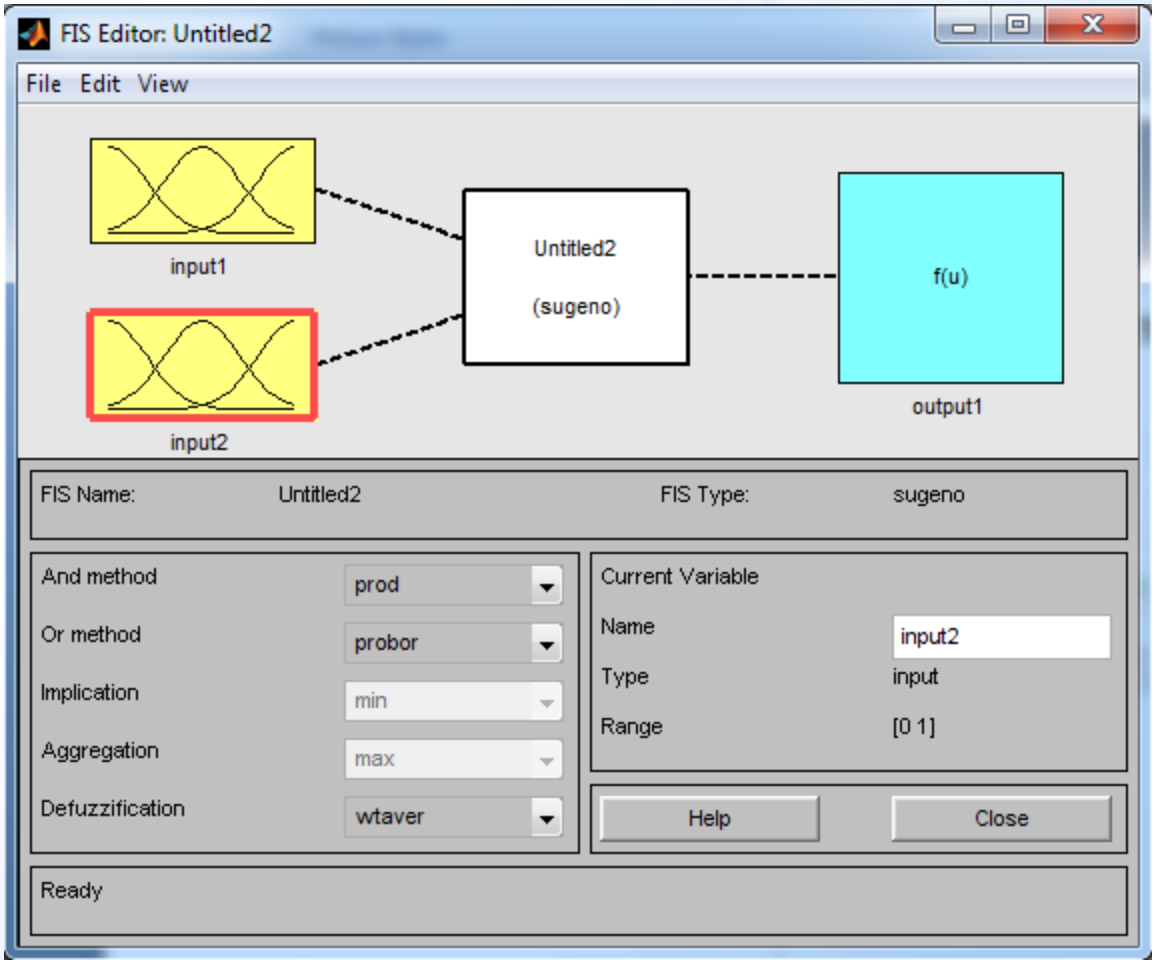
- FIS Editor



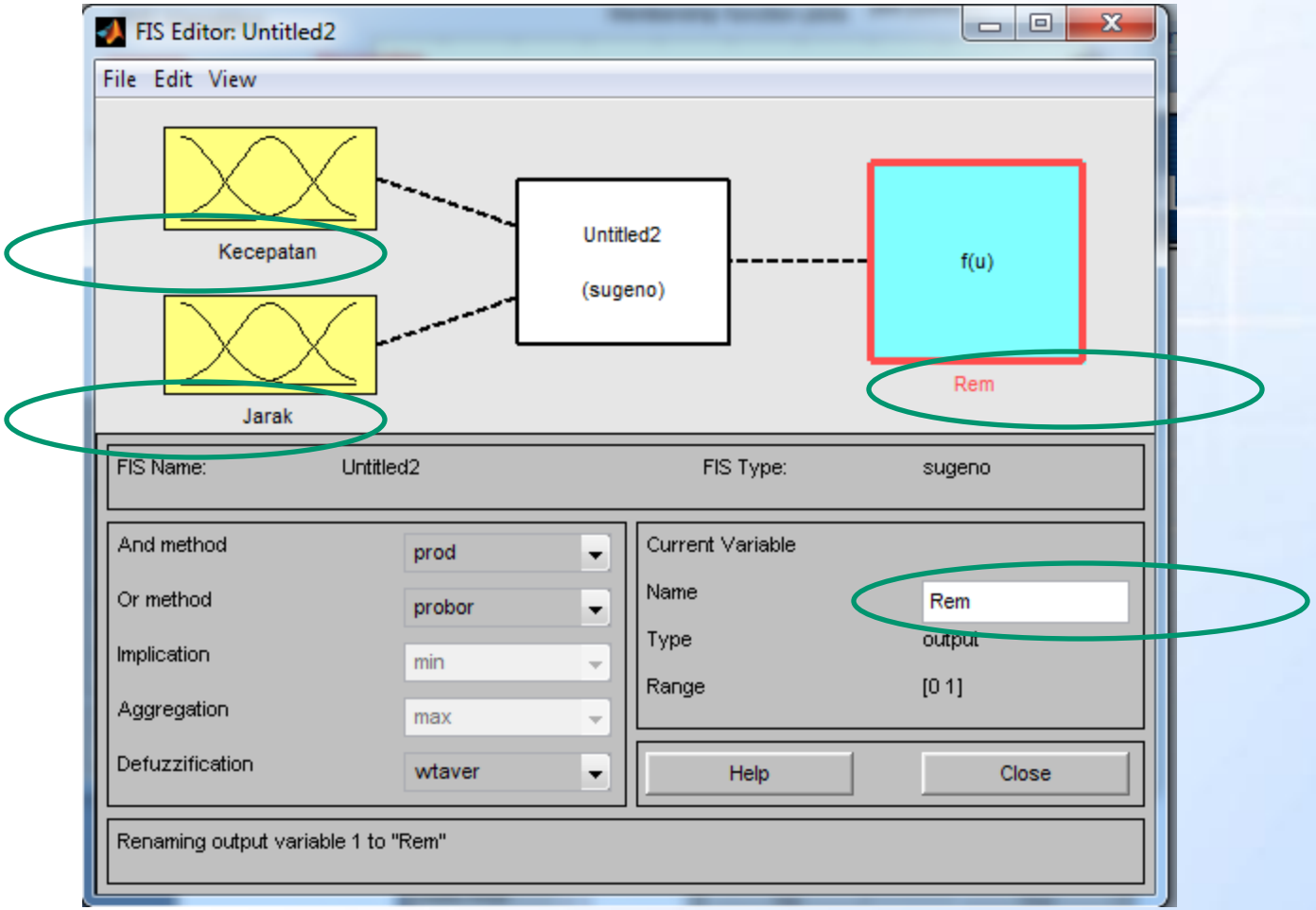

FIS Editor: Untitled
File Edit View
New FIS...
Import
Export
Print Ctrl+P
Close Ctrl+W
Mamdani Ctrl+N
Sugeno
Untitled
(mamdani


FIS Editor: Untitled2
File Edit View
Untitled2
(sugeno)
f(u)
input1
output1
FIS Name: Untitled2
FIS Type: sugeno
And method prod
Or method probor
Implication min
Aggregation max
Defuzzification wtaver
Current Variable
Name input1
Type input
Range [0 1]
Help
Close
System "Untitled2": 1 input, 1 output, and 0 rules


File
Edit
View
Undo Ctrl+Z
Add Variable...
Remove Selected Variable Ctrl+X
Membership Functions... Ctrl+2
Rules... Ctrl+3
Anfis... Ctrl+4
Input
Output
(sugeno)

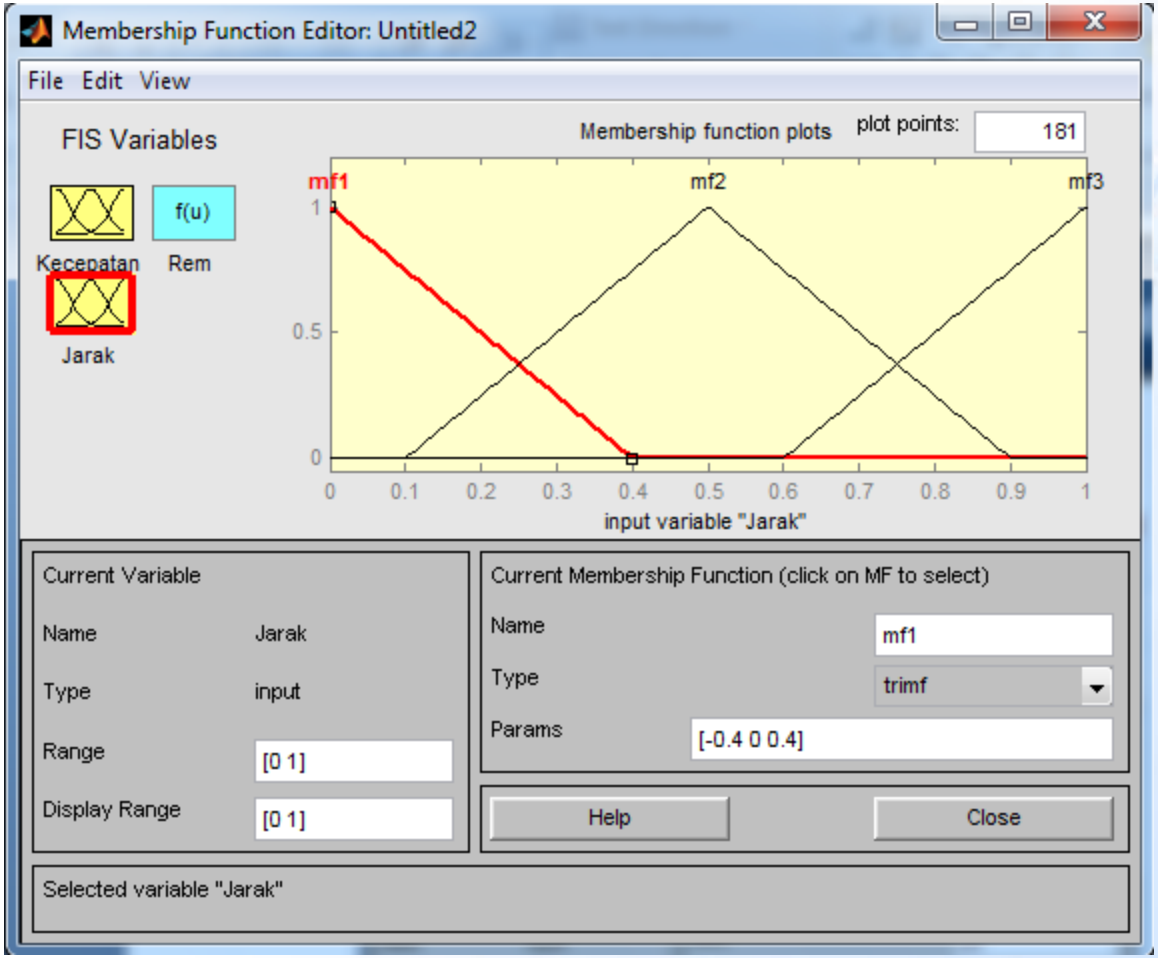

FIS Editor: Untitled2
File Edit View
input1
input2
Untitled2
(sugeno)
f(u)
output1
FIS Name: Untitled2
FIS Type: sugeno
And method prod
Or method probor
Implication min
Aggregation max
Defuzzification wtaver
Current Variable
Name input2
Type input
Range [0 1]
Help
Close
Ready

Edit nama untuk setiap kotak



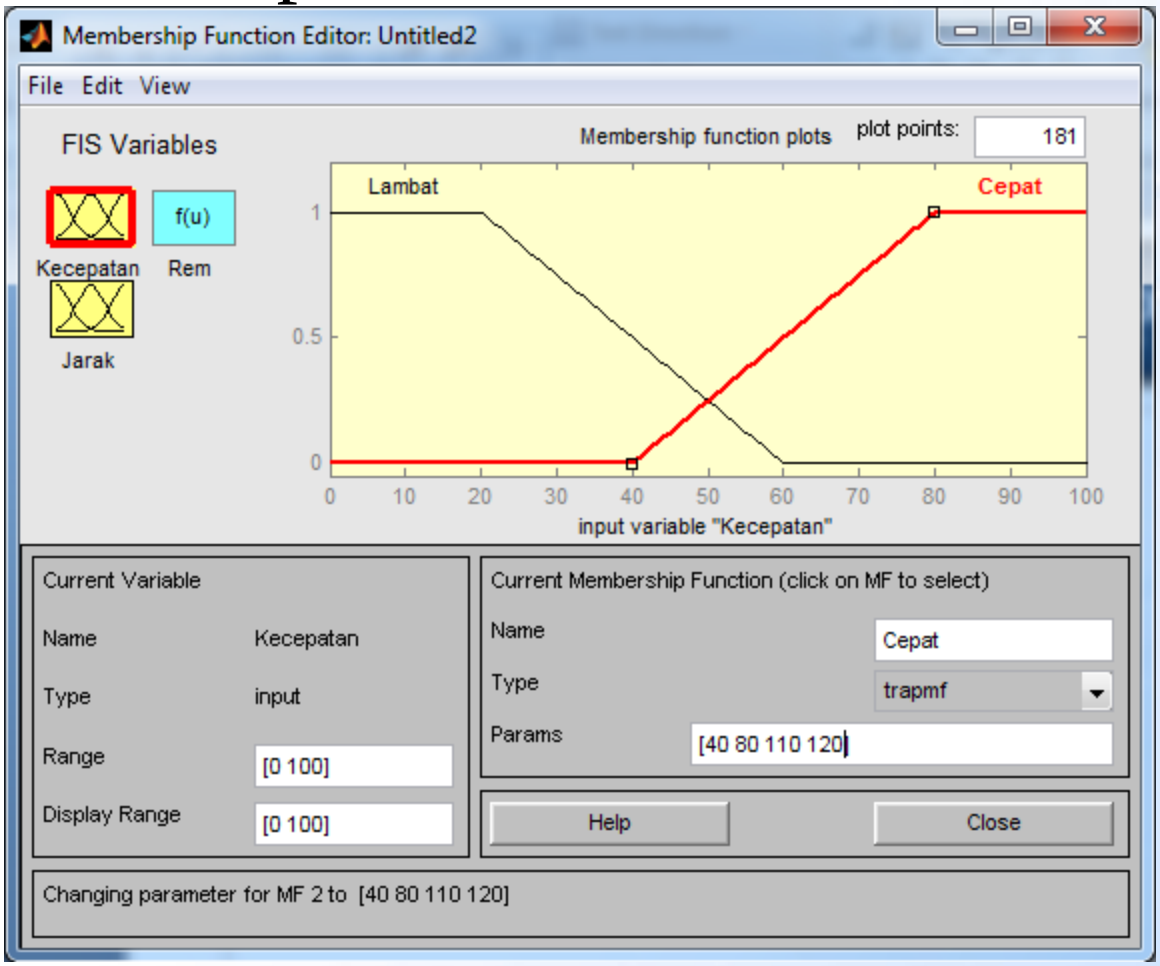
Klik 2x pada kotak input

- Akan muncul Membership Function Editor

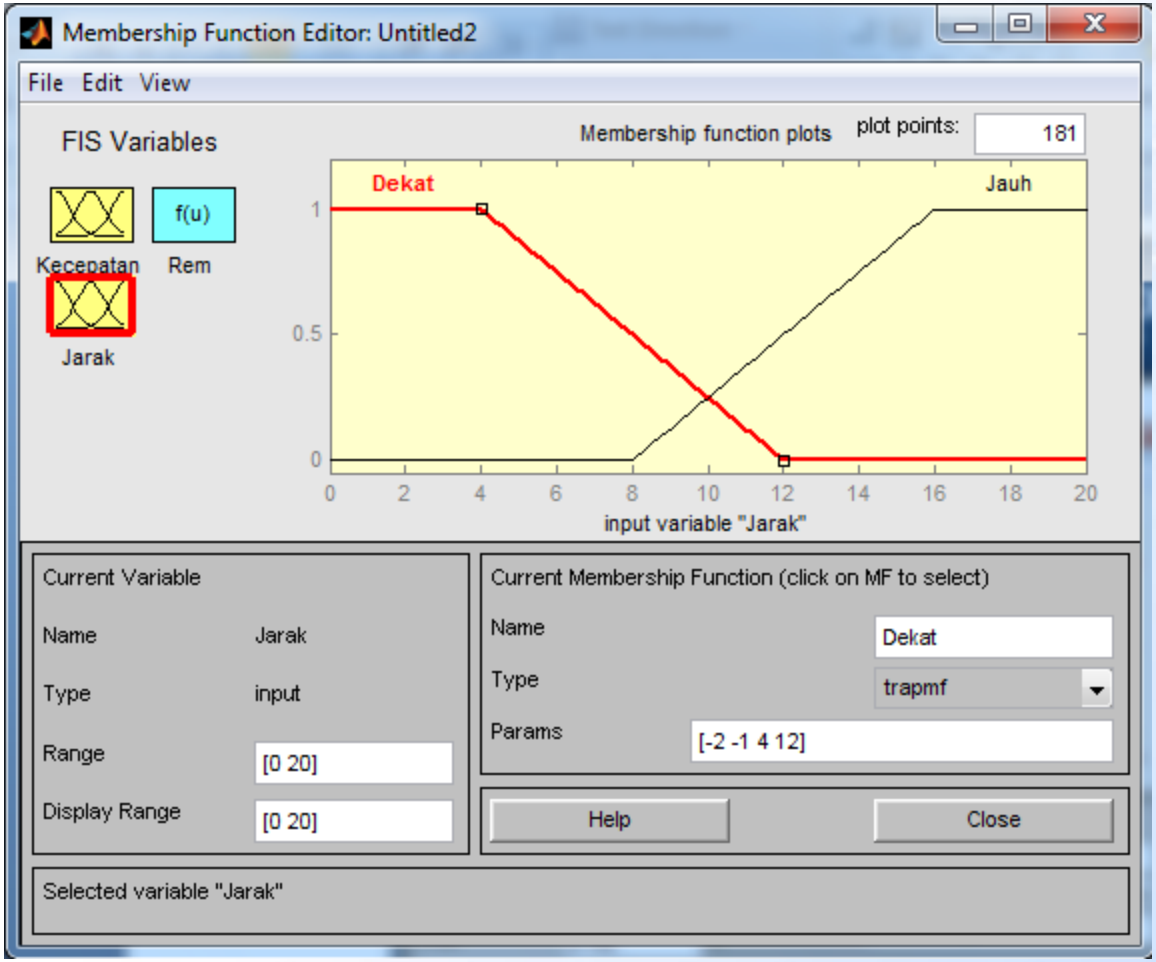


Edit Name, Range, Type, Param

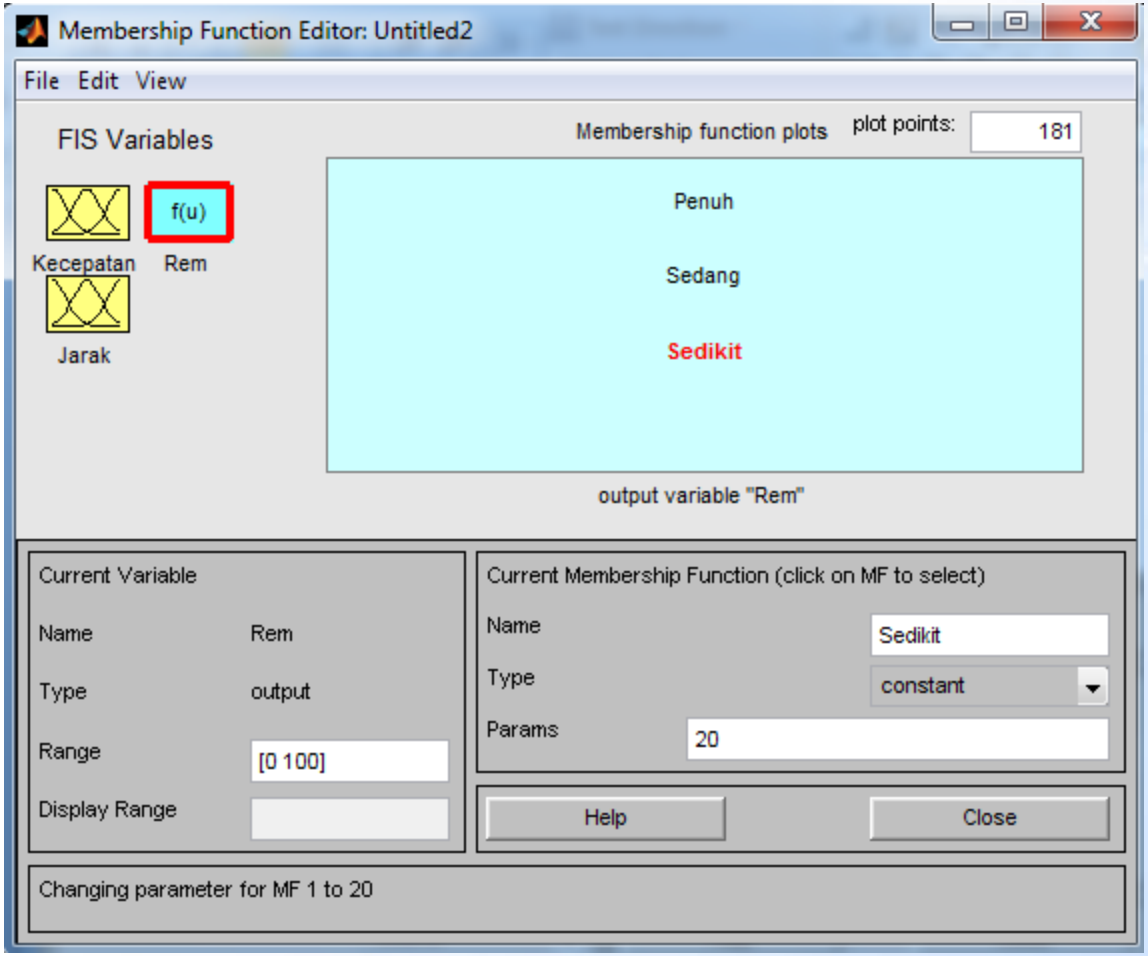
- Input : Kecepatan



- Input : Jarak

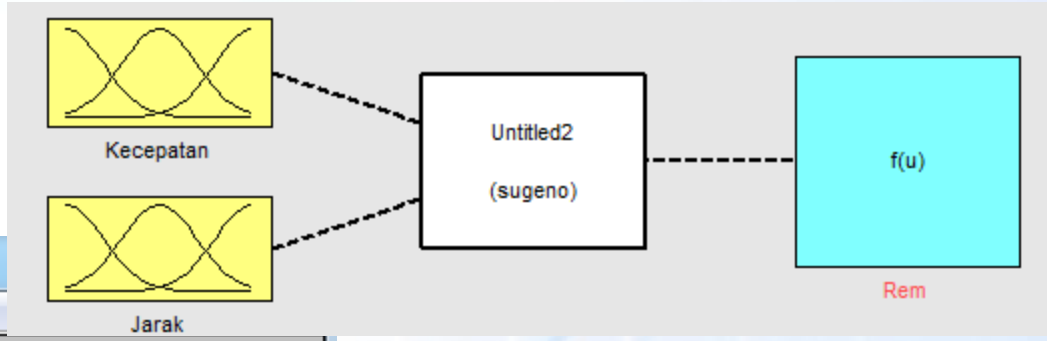

Membership Function Editor: Untitled2
File Edit View
FIS Variables
Kecepatan
f(u)
Rem
Jarak
Membership function plots
plot points:
181
Dekat
Jauh
1
0.5
0
0 2 4 6 8 10 12 14 16 18 20
input variable "Jarak"
Current Variable
Name Jarak
Type input
Range [0 20]
Display Range [0 20]
Current Membership Function (click on MF to select)
Name Dekat
Type trapmf
Params [-2 -1 4 12]
Help
Close
Selected variable "Jarak"

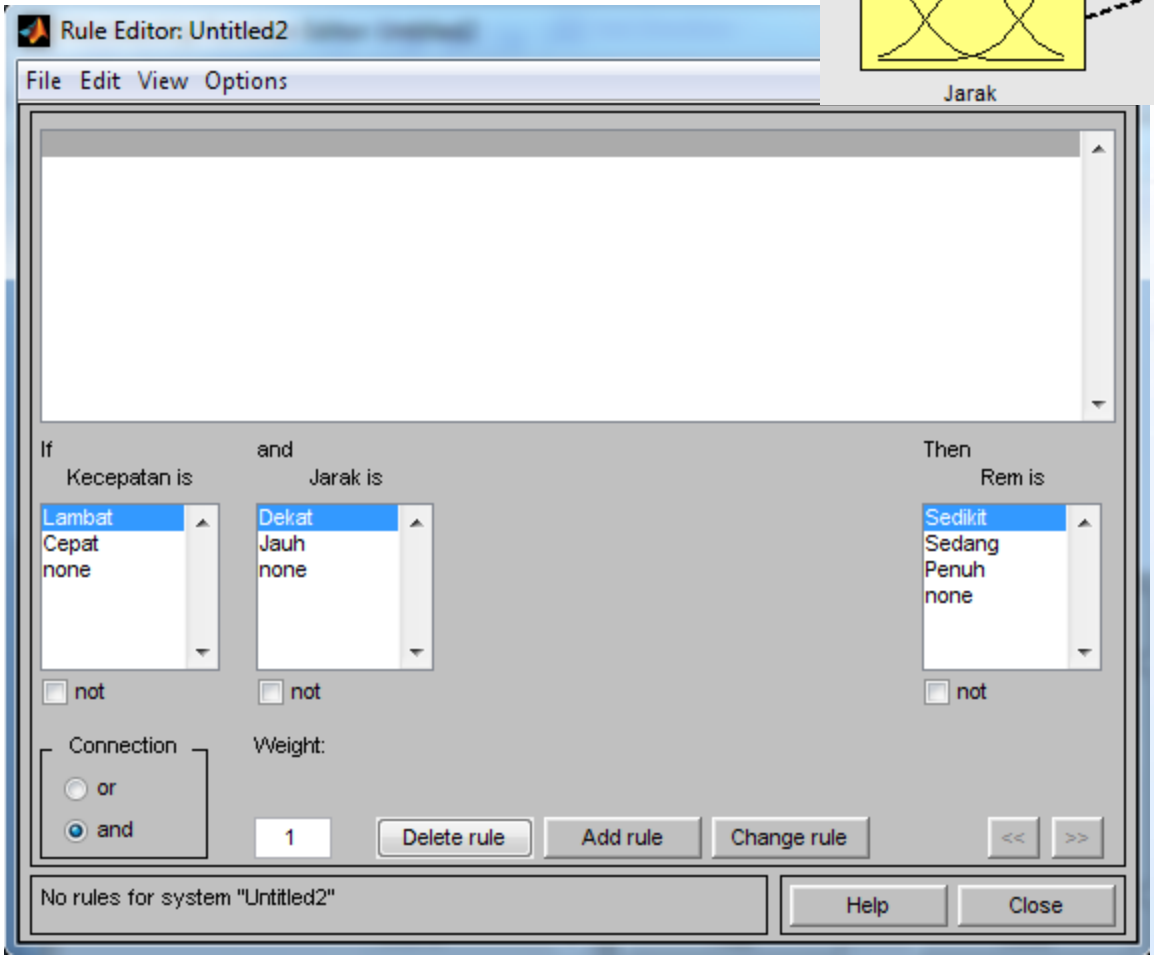
- Output : Rem


Membership Function Editor: Untitled2
File Edit View
FIS Variables
f(u)
Kecepatan
Rem
Jarak
Membership function plots
plot points:
181
Penuh
Sedang
Sedikit
output variable "Rem"
Current Variable
Name
Rem
Type
output
Range
[0 100]
Display Range
Current Membership Function (click on MF to select)
Name
Sedikit
Type
constant
Params
20
Help
Close
Changing parameter for MF 1 to 20

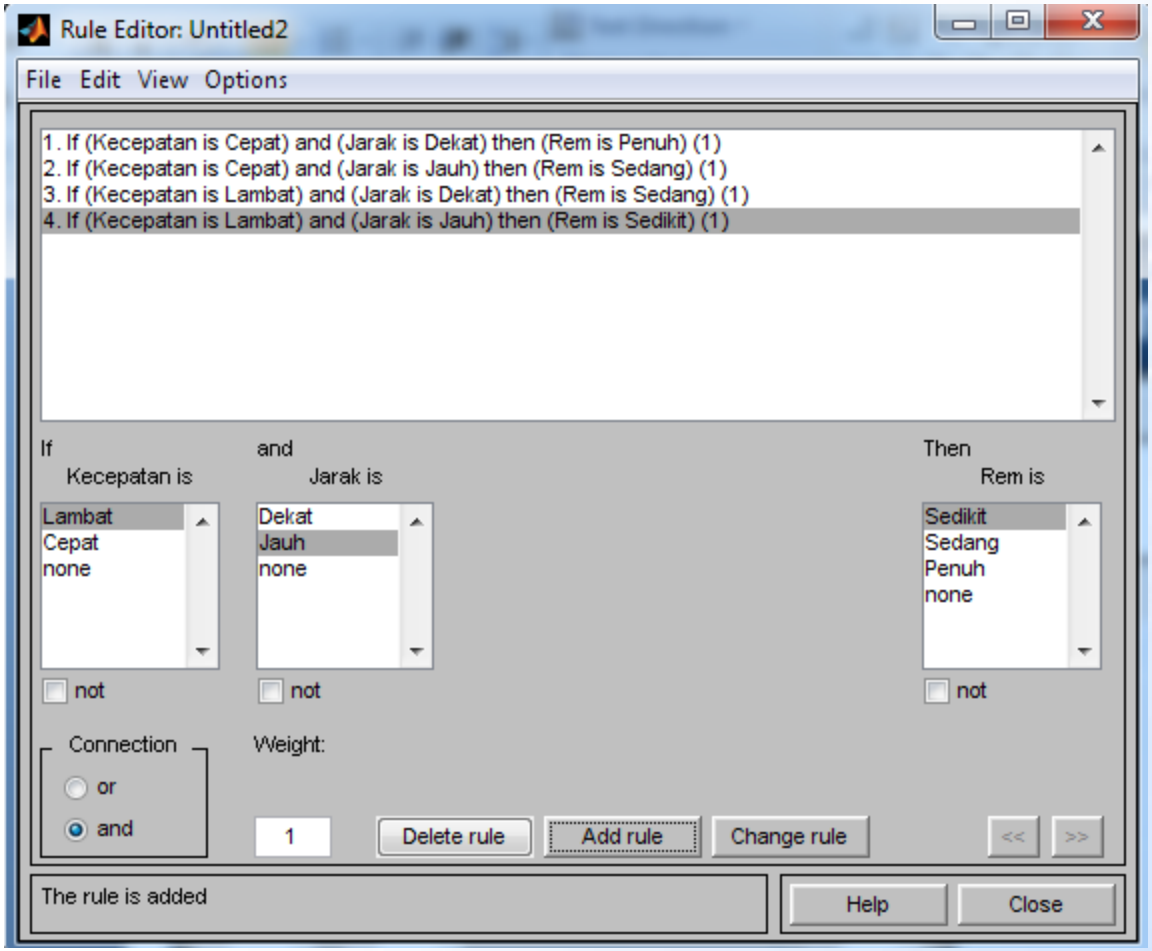
Masuk ke Rule Editor

- Klik dua kali pada kotak putih di FIS Editor

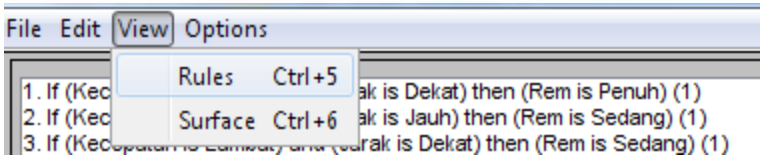




Membuat Rule

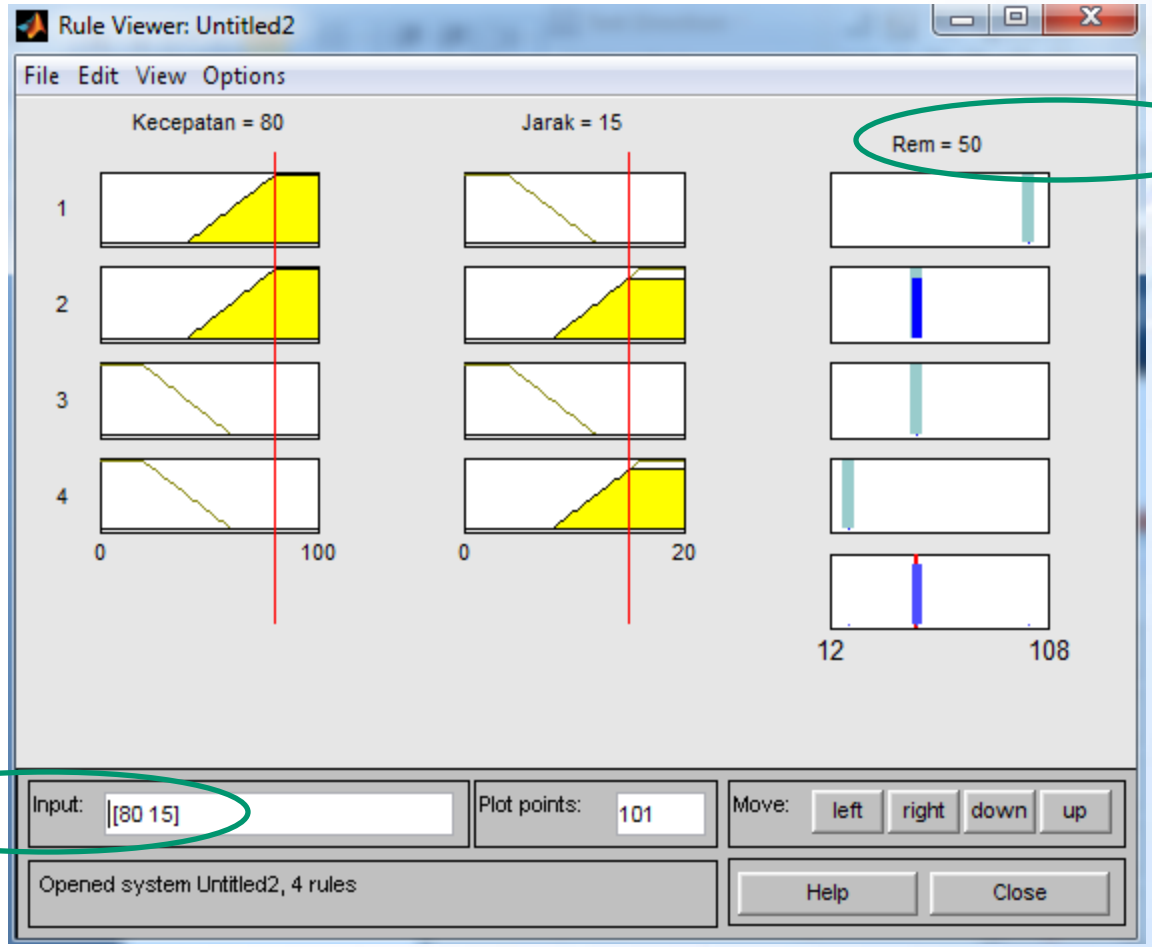


Masuk pada Rule Viewer



Simulasi input output Fuzzy

Keluaran



Masukan