**Fungsi Trigonometri Invers**

**Fungsi Invers Sinus dan Kosinus**

Definisi

Untuk memperoleh invers dari sinus dan kosinus kita batasi daerah asal fungsi-fungsi itu pada selang [-π/2,π/2] dan [0,π]

sehingga$ x=sin^{-1}y\leftrightarrow y=\sin(x dan-π/2\leq x\leq π/2)$

$$x=cos^{-1}y\leftrightarrow y=\cos(x dan 0\leq x\leq π)$$

Contoh:

$$sin^{-1}\left(^{\sqrt{2}}/\_{2}\right)=\frac{π}{4}$$

$$sin^{-1}\left(-1/2\right)=-\frac{π}{6}$$

$$cos^{-1}\left(^{\sqrt{3}}/\_{2}\right)=\frac{π}{6}$$

$$cos^{-1}\left(-^{1}/\_{2}\right)=\frac{2π}{3}$$

$$cos\left(cos^{-1}0,6\right)=0,6$$

$$sin^{-1}\left(sin\frac{3π}{2}\right)=-\frac{π}{2}$$

Definisi

Untuk memperoleh invers fungsi tangen, kita batasi daerahnya pada selang [-π/2,π/2], sehingga

$$x=tan^{-1}y\leftrightarrow y=\tan(x dan-\frac{π}{2})<x<\frac{π}{2}$$

Untuk memperoleh invers fungsi sekan, kita batasi daerahnya pada selang [0,π/2)ᴜ(π/2,π], sehingga

$$x=sec^{-1}y\leftrightarrow y=sec x dan 0\leq x\leq π, dengan x\ne \frac{π}{2}$$

$$sec^{-1}y=cos^{-1}\left(\frac{1}{y}\right)$$

Hitunglah

$$tan^{-1}\left(1\right)=π$$

$$tan^{-1}\left(-\sqrt{3}\right)=-\frac{π}{3}$$

$$tan^{-1}\left(-0,145\right)=-0,144$$

$$sec^{-1}\left(-1\right)=cos^{-1}\left(-1\right)=π$$

$$sec^{-1}\left(2\right)=cos^{-1}\left(\frac{1}{2}\right)=\frac{π}{3}$$

Empat Pemakaian Kesamaan

$$\sin(\left(cos^{-1}x\right))=\sqrt{1-x^{2}}$$

$$cos\left(sin^{-1}\right)=\sqrt{1-x^{2}}$$

$$sec\left(tan^{-1}x\right)=\sqrt{1+x^{2}}$$

$$\tan(\left(sec^{-1}x\right)=\pm \sqrt{x^{2}-1})$$

Contoh

1. Hitunglah $\sin(\left[2cos^{-1}\left(\frac{2}{3}\right)\right])$

Jawab:

Ingat sudut ganda sin 2θ = 2 sin θ cos θ

$$sin\left[2 cos^{-1}\left(\frac{2}{3}\right)\right]=2\sin(\left[cos^{-1}\left(\frac{2}{3}\right)\right]\cos(\left[cos^{-1}\left(\frac{2}{3}\right)\right]))$$

$$=2.\sqrt{1-\left(\frac{2}{3}\right)^{2}}.\frac{2}{3}=\frac{4\sqrt{5}}{9}$$

1. Perlihatkan bahwa $\cos(\left(2tan^{-1}x\right)=\frac{1-x^{2}}{1+x^{2}})$

Jawab:

Ingat sudut ganda cos 2θ= 2 cos2θ-1

 $\cos(\left(2tan^{-1}x\right)=cos2θ)=2cos^{2}θ-1=\frac{2}{sec^{2}θ}-1$

$$=\frac{2}{1+tan^{2}θ}-1=\frac{2}{1+x^{2}}-1=\frac{1-x^{2}}{1+x^{2}}$$

Latihan:

Tentukan nilai yang tepat

1. $sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
2. $\arccos(\left(x\right))=cos^{-1}x$

$$arccos⁡(-\frac{\sqrt{2}}{2})$$

1. $tan^{-1}\left(-\sqrt{3}\right)$
2. $sec^{-1}\left(-2\right)$
3. $sin^{-1}\left(-1\right)$
4. $\sin((sin^{-1}0,541))$
5. $\cos(\left(\arctan(3,125)\right))$
6. $cos\left[2sin^{-1}\left(-\frac{2}{3}\right)\right]$
7. $sin\left[cos^{-1}\left(\frac{3}{5}\right)+cos^{-1}\left(\frac{5}{13}\right)\right]$