1. $y_0 = 10.000$
$t = 10$
$y = 24.000$

$y = y_0 e^{kt}$
$24\cancel{000} = 10\cancel{000}\, e^{k10}$
$\frac{24}{10} = e^{10k}$
$\ln \frac{24}{10} = 10k$
$k = \frac{\ln 2,4}{10} = 0,08755$

$y = y_0 e^{kt}$
$= 10.000\, e^{0,08755 . 25}$
$= 10.000\, e^{2,18867}$
$= 89,233,536$

∴ setelah 25 hari bakteri menjadi 89.300 //

2. $y = y_0 e^{kt}$
$20.000 = 10.000\, e^{\ln \frac{2,4}{10} t}$
$2 = e^{\ln \frac{2,4}{10} t}$
$\ln 2 = 0,08755 . t$
$t = \frac{\ln 2}{0,08755} = 7,917 \approx 8$

∴ Setelah 8 hari bakteri bertambah menjadi 2x

3. $\cancel{t} = 1/2$. $t = 810$
$\cancel{t}_0 = 1$
$y = y_0 e^{-kt}$
$1/2 = 1\, e^{-k.810}$
$1/2 = e^{-k810}$
$\ln 2^{-1} = k\, 810$
$k = \frac{\ln 2^{-1}}{810} = -0,000856$

$y = y_0 e^{kt}$
$= 10\, e^{-0,000856 . 300}$
$= 7,7352$

∴ sisa zat setelah 300 tahun = 7,73 gr

4. $y = y_0 e^{kt}$
$1/2 = 1\, e^{k10}$
$\ln 2^{-1} = 10k$
$k = \frac{-\ln 2}{10} = -0,0693$

$y = y_0 e^{kt}$
$1 = 100\, e^{-0,0693\, t}$
$\ln \frac{1}{100} = -0,0693\, t$
$\ln 10^{-2} = -0,0693\, t$
$\frac{-2 \ln 10}{-0,0693} = 66,4527$
≈ 66

∴ Zat radioaktif akan meluluh setelah 66 tahun.

Bunga Majemuk $A = A_0 \left(1+\frac{r}{n}\right)^{nt}$

1) a. 375.000

$t = 2$

$r = 9{,}5\%$

$A(1) = 375.000 \left(1 + \frac{0{,}095}{1}\right)^{1 \cdot 2}$

$= 375.000 \cdot (1{,}095)^2$

$= 449.634 \approx 450.000$

b. $A(12) = 375.000 \left(1 + \frac{0{,}095}{12}\right)^{12 \cdot 2}$

$= 375.000 \; (1{,}20835)$

$= 453.129$

≈ 453.000

c. $A(365) = 375.000 \left(1 + \frac{0{,}095}{365}\right)^{365 \cdot 2}$

$= 453.457 \approx 454.000$

d. $A = A_0 e^{rt}$

$= 375.000 \; e^{0{,}095 \cdot 2}$

$= 375.000 \; e^{0{,}19}$

$= 453.468{,}6 \approx$

2). Sejumlah uang 2x lipat

$A = A_0 \left(1 + \frac{r}{n}\right)^{nt}$

$2 = 1 \left(1 + \frac{0{,}12}{12}\right)^{12t}$

$2 = 1{,}01^{12t}$

$\ln 2 = 12t \ln 1{,}01$

$\frac{\ln 2}{\ln 1{,}01} = 12t$

$t = \frac{\frac{\ln 2}{\ln 1{,}01}}{12} = 5{,}8 \approx 6$

$A = A_0 e^{rt}$

$2 = 1 e^{0{,}12t}$

$\ln 2 = 0{,}12t$

$t = \frac{\ln 2}{0{,}12} = 5{,}78 \approx 6$

Lat. fs. Trigonometri Invers

1. $\sin^{-1}\left(\frac{1}{2}\sqrt{3}\right) = \frac{\pi}{3}$

2. $\arccos(x) = \arccos\left(-\frac{1}{2}\sqrt{2}\right)$

$= \frac{3}{4}\pi$

3. $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

4. $\sec^{-1}(-2) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

5. $\sin^{-1}(1) = \frac{\pi}{2}$

6 $\sin(\sin^{-1} 0{,}541) = 0{,}541$

7. $\cos(\arctan 3{,}125) = 0{,}30478$

8. $\cos\left[2\sin^{-1}\left(-\frac{2}{3}\right)\right] \rightarrow \theta = \sin^{-1}\left(-\frac{2}{3}\right)$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\sin^2\left(\sin^{-1}\left(-\frac{2}{3}\right)\right)$

$= 1 - 2\left[\sin\left(\sin^{-1}\left(-\frac{2}{3}\right)\right) \sin\left(\sin^{-1}\left(-\frac{2}{3}\right)\right)\right]$

$= 1 - 2\left[-\frac{2}{3} \cdot -\frac{2}{3}\right] = 1 - 2 \cdot \frac{4}{3}$

$= 1 - \frac{8}{3} = -\frac{5}{3}$ //

9. $\sin\left[\cos^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right]$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \sin.\cos^{-1}\left(\frac{3}{5}\right) \cos.\cos^{-1}\left(\frac{5}{13}\right) +$

$\cos.\cos^{-1}\left(\frac{3}{5}\right).\sin\cos^{-1}\left(\frac{5}{13}\right)$

$= \sqrt{1-\left(\frac{3}{5}\right)^2} \cdot \frac{5}{13} + \frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}$

$= \sqrt{1-\frac{9}{25}} \cdot \frac{5}{13} + \frac{3}{5}\sqrt{1-\frac{25}{169}}$

$= \sqrt{\frac{16}{25}} \cdot \frac{5}{13} + \frac{3}{5}\sqrt{\frac{144}{169}} = \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13}$

$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$ //

Lat Turunan fs trigonometri

Tentukan dy/dx

1. $y = \cos^2(x-2)$

$y' = 2\cos(x-2) \cdot D_x(\cos(x-2)) \cdot D_x(x-2)$

$= 2\cos(x-2) \cdot \sin(x-2) \cdot 1$

$= 2\sin(x-2)\cos(x-2)$ //

2. $y = \sin\sqrt{2+7x}$

$y = \sin(2+7x)^{1/2}$

$y' = D_x \sin(2+7x)^{1/2} \cdot D_x(2+7x)^{1/2} \cdot D_x(2+7x)$

$= \cos(2+7x)^{1/2} \cdot \frac{1}{2}(2+7x)^{-1/2} \cdot 7$

$= \dfrac{7\cos\sqrt{2+7x}}{2\sqrt{2+7x}}$

3. $y = \cot x \operatorname{cosec} x$

$y' = uv' + vu'$

$u = \cot x$

$u' = -\operatorname{cosec}^2 x$

$v = \operatorname{cosec} x$

$v' = -\operatorname{cosec} x \cot x$

$y' = \cot x \cdot (-\operatorname{cosec} x \cot x) +$

$\operatorname{cosec} x \cdot (-\operatorname{cosec}^2 x)$

$= -\operatorname{cosec} x \cdot \cot^2 x + (-\operatorname{cosec}^3 x)$

$= -\operatorname{cosec} x (\cot^2 x + \operatorname{cosec}^2 x)$

$= -\operatorname{cosec} x (\operatorname{cosec}^2 x - 1 + \operatorname{cosec}^2 x)$

$= -\operatorname{cosec} x (2\cos^2 x - 1)$

$= \operatorname{cosec} x \; x - 2\cos^3 x$ //

4. $y = e^{\cot x}$

$y' = e^{\cot x} \cdot D_x \cot x$

$= e^{\cot x} \cdot (-\operatorname{cosec}^2 x)$

$= -\operatorname{cosec}^2 x \cdot e^{\cot x}$

5. $y = \sin^{-1}(x^2)$

$y' = \dfrac{1}{\sqrt{1-(x^2)^2}} = \dfrac{2x}{\sqrt{1-x^4}}$

6. $y = e^x \cos^{-1}\sqrt{x}$

$u = e^x \quad u' = e^x$

$v = \cos^{-1}\sqrt{x} \quad v' = \dfrac{-1}{\sqrt{1-(\sqrt{x})^2}} = \dfrac{-1}{\sqrt{1-x}}$

$y' = uv' + vu'$

$= e^x\left(\dfrac{-1}{\sqrt{1-x}}\right) + \cos^{-1}\sqrt{x} \cdot e^x$

$= e^x\left[\dfrac{-1}{\sqrt{1-x}} + \cos^{-1}\sqrt{x}\right]$

7. $\int x \sin(x^2)\, dx$ $\quad u = x^2 \quad du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$= \int \sin u \cdot \frac{1}{2}\, du$

$= \frac{1}{2} \int \sin u\, du$

$= \frac{1}{2}(-\cos u) + c \quad = \frac{-\cos x^2}{2} + c$

8. $\int \frac{\sec^2 x}{\tan x}\, dx$ $\quad u = \tan x \quad du = \sec^2 x\, dx$

$= \int \frac{du}{u} = \ln|u| + c = \ln|\tan x| + c$

9. $\int_0^{1/2\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x \Big]_0^{1/2\sqrt{2}}$

$= \sin^{-1} \frac{1}{2}\sqrt{2} - \sin^{-1} 0$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$ //

10. $\int \frac{1}{1+4x^2}\, dx = \int \frac{1}{1+(2x)^2}\, dx$ $\quad u = 2x$

$du = 2\, dx$

$= \int \frac{1}{1+u^2} \cdot 1/2\, du \quad \frac{1}{2} du = dx$

$= \tan^{-1} u + c$

$= \tan^{-1} 2x + c$

=

Lat. Teknik Pengintegralan

1. $\int (x-1)^4 dx$ $\qquad u = x-1 \quad du = dx$

$= \int u^4 \, du = \frac{1}{5} u^5 + C = \frac{1}{5}(x-1)^5 + C$

2. $\int x(x^2+1)^4 \, dx$ $\qquad u = x^2+1 \quad du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$\int u^4 \frac{1}{2} du = \frac{1}{2} \int u^4 du$

$= \frac{1}{2} \cdot \frac{1}{5} u^5 + C = \frac{1}{10}(x^2+1)^5 + C$

3. $\int \frac{dx}{x^2+1} =$

$\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$

$\int \frac{dx}{1+x^2} = \frac{1}{1} \tan^{-1}\left(\frac{x}{1}\right) + C = \tan^{-1}(x) + C$

4. $\int \frac{y}{\sqrt{16-9y^4}} \, dy$ $\qquad u = 3y^2 \quad du = 6\boxed{y \, dy}$

$\frac{1}{6} du = y \, dy$

$= \int \frac{\frac{1}{6} du}{\sqrt{4^2-(3y^2)^2}} = \frac{1}{6} \int \frac{du}{\sqrt{4^2-(3y^2)^2}}$

$= \frac{1}{6} \sin^{-1}\left(\frac{3y^2}{4}\right) + C$

5. $\int \frac{dt}{t\sqrt{2t^2-9}}$ $u = \sqrt{2}t$ $du = \sqrt{2}.\, dt$

$\int \frac{du}{u\sqrt{u^2-9^2}} = \frac{1}{9} \sec^{-1}\left(\frac{|u|}{9}\right) + C$

$= \frac{1}{3} \sec^{-1}\left(\frac{\sqrt{2}|t|}{3}\right) + C$ //