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Comparison of Reconstruction Algorithm on Sparse Representation based Classification (SRC) for Face Recognition

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***Abstract*—Sparse representation based Classification (SRC) has gained the attention of pattern recognition and computer vision researchers, especially researchers working on face recognition. On SRC's algorithm, it is necessary to find a solution to an optimization problem to recover $\mathbf{x}$ from the equation $\mathbf{y} = \mathbf{A}\mathbf{x}$. Only a few studies reported the reconstruction of the signals on SRC's algorithm. Therefore, this paper studies the comparison of OMP, LASSO, and CVX to help the readers understand the reconstruction algorithm's effect on SRC. The simulation result is that LASSO and CVX algorithms have the same recognition rate, but LASSO can compute twice faster as CVX. On the other hand, the OMP algorithm can give the highest recognition rate on a specific dimension of the image with a faster computation time than LASSO.**

***Index Terms*—SRC, Sparse Representation, Reconstruction**

I. Introduction

Over the past few decades, Compressive Sensing (CS) has emerged as one of the fascinating areas of study in signal processing and optimization. Donoho originally applied this theory in 2006 [1], then popularized by Candés *et al*. in 2008 [2]. Shannon theory-based conventional signal acquisition methods are being challenged by the revolutionary way that CS has changed the paradigm for sensing or sampling [3]. Numerous natural signals are either sparse or can be compressed given the appropriate basis [4].

Numerous aspects of signal processing, require the solution of a sparse approximation problem such as denoising [5], image-inpainting [6], target detection [7], computer vision [8] and pattern recognition [9], etc. Sparse representation refers to solving the system of equations $\mathbf{y} = \mathbf{A}\mathbf{x}$ when the matrix $\mathbf{A}$ has more columns than rows, and the vector $\mathbf{x}$ is sparse. They must recover a sparse signal from a collection of undersampled measurements. There are many sparse recovery algorithms have been proposed. Non-convex optimization techniques, convex relaxations, and greedy algorithms are the most common types of sparse recovery algorithm [10]. Fig. 1 depicted the classification of the sparse recovery algorithm based on these categories.

Two well-known methods for requiring sparsity in the solution are the ℓ_0-quasinorm (number of components in the vector that are not zero), which results in an implausibly challenging numerical problem, and the ℓ_1-norm. It is common knowledge that applying a regularization term like the ℓ_0-quasinorm is necessary to recover vectors with more nonzero coefficients than the ℓ_1-norm. Standard convex optimization techniques can be used to solve the ℓ_1-min problem [11]

Finding sparse solutions has become increasingly important in computer vision, pattern recognition, and image analysis. In particular, in the context of Face Recognition (FR), the primary objective of determining a person's identity based on an image of their face is given a collection of example faces. The sparse representation-based classification (SRC) suggested by Wright *et al.* provides a robust answer for FR problems. Such

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as dimensionality reduction using the downscale technique, handling occlusion, and image corruption [12].

The SRC technique is based on the fundamental idea that other examples of the same class can linearly represent an image of a face. Linearly, each class is distinct from the others. A face data set is a collection of images of people's faces that are organized into a matrix with the notation $\mathbf{A} \in \mathbf{R}^{w \times h}$ as a representation of the data training samples. In numerous image processing techniques, the vector representation version of the matrix $\mathbf{A}$ is denoted by $\mathbf{v} \in \mathbf{R}^m$, and $m = w \times h$, where w and h respectively represent the width and height of the face image. The accuracy in FR problems is determined by calculating this $\mathbf{x}$ value. The desired solution $\mathbf{x}$ is as sparse as possible. The majority of SRC's modification algorithms employ ℓ_1-norm minimization. However, to classify the test image more accurately, we must use the most sparse value of $\mathbf{x}$.

In this paper, we study and simulated the reconstruction algorithm based on the ℓ_1-norm minimization using convex optimization and ℓ_0-norm minimization using the greedy algorithm. We compared the performance based on the accuracy and computation time. As far as we know, the SRC method's comparison of ℓ_0-norm and ℓ_1-norm reconstruction is not yet available.

- Sparse Recovery Algorithms
  - Convex Relaxation
    - Basis Pursuit (BP)
    - Least Absolute Shrinkage & Selection Operator (LASSO)
    - Least Angle Regression (LARS)
    - Approximate Message Passing (AMP)
    - Gradient Descent with Sparsification (GraDes)
    - Iterative Soft Thresholding (IST)
  - Non-Convex Optimization
    - Bayesian Compressive Sensing (BCS)
    - Focal Underdetermined System Solution (FOCUSS)
    - Iterative Reweighted Least Squares (IRLS)
  - Greedy Algorithm
    - Matching Pursuit (MP)
    - Matching Pursuit based on Least Squares (MPLS)
    - Orthogonal Matching Pursuit (OMP)
    - Subspace Pursuit (SP)
    - Stagewise Orthogonal Matching Pursuit (SOMP)
    - Compressive Sampling Matching Pursuit (CoSaMP)
    - Regulatized OMP (ROMP)
    - Generalized Orghogonal Matching Pursuit (GOAMP)
    - Gradient Pursuit (GP)
    - Multipath Matching Pursuit (MMP)

Fig. 1. Classification of Sparse Recovery Algorithm Adopted From [10]

II. Mathematical Optimization

SRC recognition method belongs to mathematical optimization problem. A mathematical problem formulation, also known as an optimization problem, presented in this form [13]:

$$\begin{aligned} \text{minimize} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le b_i, i = 1, ..., m \end{aligned} \tag{1}$$

where $x = (x_1, x_2, ..., x_n)$ is the optimization variables, $f_0 : \mathbf{R}^n \to \mathbf{R}$ is the objective function and $f_i : \mathbf{R}^n \to \mathbf{R}, i = 1, 2, ...m$ is the constraint functions, and the constants $b_1, ..., b_m$ are the limits, or bounds, for the constraints. The optimal solution of $\hat{x}$ has smallest value of f_0 among all vectors that satisfy the constraints. The optimization problems, in general, are generally difficult to solve, and many of the proposed solutions come with undesirable trade-offs—for example, extremely lengthy calculation times or an inability to reliably locate the optimal answer. However, there are some problem classes that can be handled in an effective manner and with a high degree of reliability by employing techniques such as least-squares, linear programming, and convex optimization.

A. Least-Squares

The aim of a problem known as the least-squares problem is the sum of squares of terms represented by the form $a_i^T x - b_i$ [13]. This type of optimization issue does not involve any constraints.

$$\text{minimize} ||Ax - b||_2^2 = \sum_{i=1}^{k} (a_i^T x - b_i)^2 \tag{2}$$

where $A \in \mathbf{R}^{m \times n}$ (with $m \ge n$), a_i^T are the rows of A, and the vector $x \in \mathbb{R}^n$ is the optimization variable. The solution of problem (2):

$$\begin{aligned} (A^T A)x &= A^T b \\ x &= (A^T A)^{-1} A^T b \end{aligned} \tag{3}$$

In a time approximately proportional to $n^2 k$, the least-squares problem can be solved.

B. Linear Programming

Linear programming is another major sub-field of optimization problems; in this type of issue, both the constraint functions and the objective functions are linear.

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & a_x^T \le b_i, i = 1, ..., m \end{aligned} \tag{4}$$

where the vectors $x, a_1, ..., a_m \in \mathbf{R}^n$ and scalars $b_1, ..., b_m \in \mathbf{R}$ are problem parameters that specify the objective and constraint functions respectively. It is not possible to solve a linear program using a straightforward analytical method in the same way that a least-squares problem may be solved. However, there are many very effective ways to solve linear programs, such as Dantzig's simplex method.

C. Convex Optimization

One of the following formulations can represent a convex optimization problem [13]:

$$\begin{aligned} \text{minimize} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le b_i, i = 1, ..., m \end{aligned} \tag{5}$$

where the functions $f_0, ..., f_m : \mathbf{R}^n \to \mathbf{R}$ are convex, satisfy:

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y) \tag{6}$$

for all $x, y \in \mathbf{R}^n$ and all $\alpha, \beta \in \mathbf{R}$ with $\alpha + \beta = 1, \alpha \ge 0, \beta \ge 0$.

III. Sparse Representation based Classification

The robust FR using sparse representation was written by Wright *et al.* as follows [12]:

$$\mathbf{y} = \mathbf{A}\mathbf{x} \tag{7}$$

In the case of classification on face recognition, $\mathbf{y} \in \mathbf{R}^{M\times 1}$ represented the image that needed to be identified, $\mathbf{A} \in \mathbf{R}^{M\times N}$ is training sample column-wise database matrix and $\mathbf{x} \in \mathbf{R}^{N\times 1}$ is a sparse matrix. If only $K(K \ll N)$ elements of $\mathbf{x}$ are non zero and the rest elements in $\mathbf{x}$ are zero, we call the signal $\mathbf{y}$ is K-sparse. We want to find $\mathbf{x}$ if $\mathbf{y}$ and $\mathbf{A}$ are known. The sparsest solution $\hat{\mathbf{x}}$ is, there fore given by problem (8):

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_0 \quad \text{s.t} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \tag{8}$$

Despite the fact that this problem is NP-hard, it is possible to solve it using greedy methods under certain conditions (depending on the values of N, M, K, and $\mathbf{A}$) [14].

Recent studies have demonstrated that if a representative solution is produced by applying the ℓ_1-norm minimization with adequate sparsity, then the solution can be equal to the solution obtained by ℓ_0-norm with a high probability [15]. In addition, the problem of ℓ_1-norm optimization has an analytical solution and can be handled in polynomial time. As a result, extensive sparse representation approaches that incorporate ℓ_1-norm minimization have been presented to enhance the theory of sparse representation. The most common and popular structures of sparse representation with the ℓ_1-norm minimization, which are very similar to sparse representation with the ℓ_0-norm minimization, are typically employed to solve the following problems:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_1 \quad \text{s.t} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \tag{9}$$

In the case of noisy measurement, the optimization problem given in problem (9) is relaxed to the following problem [15]:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_1 \quad \text{s.t} \quad ||\mathbf{A}\mathbf{x} - \mathbf{y}||_{\mathbf{2}}^{\mathbf{2}} \leq \varepsilon \tag{10}$$

The test image $\mathbf{y}$ is classify based on the approximation by assigning it to the class of object with minimum residual between $\mathbf{y}$ and $\hat{\mathbf{y}}$:

$$\min_i r_i(\mathbf{y}) = ||\mathbf{y} - A\delta_i(\hat{\mathbf{x}})||_2 \tag{11}$$

Wright *et al.* studied the implication of feature extraction, which carries over the SRC framework, by reducing the dimensionality of data and computational cost [12]. One key element in the practical application of SRC is the dimension reduction of the training samples. The number of calculations directly influences the initial size of the training samples, affecting the algorithm's complexity. This dimension reduction is also perceived as feature extraction from original sample images. A formula for reduction factor (ρ) from the raw image's $\mathbb{R}^M$ to a lower-dimensional feature $\mathbb{R}^d$ ($d << M$) as follows:

$$\rho = \frac{\text{M (raw image)}}{\text{d (reduced image)}} \tag{12}$$

IV. Greedy Approach

The task of addressing sparse representation with ℓ_0-norm regularization, often known as the problem (8), is considered an NP-hard problem. The greedy strategy offers a method for obtaining an approximate solution for sparse representation problems. In reality, the greedy approach cannot directly resolve the optimization problem; all it can do is search for an approximation of a solution to a problem (8). Greedy algorithms use the notion of dictionary atomic matching to seek the optimal global solution from the local optimal. This algorithm is utilized by gradually raising the practical column to achieve the closest answer to the initial signal. The empty set serves as the starting point for the practical set, which is then updated column by column by locating the minimum reconstruction residual. The operation is complete when the residual is below a predetermined threshold.

Two examples of typical sparse representation greedy algorithms are The matching pursuit (MP) and the orthogonal matching pursuit (OMP) [16]. Both MP and OMP have speedy recovery times and inexpensive implementation costs. The properties of the two greedy algorithms are contrasted and compared in the Table I [17].

TABLE I
Comparison of Greedy Algorithms [16]

Algorithm Name	Samples of Observations	Complexity
MP	$K\log_2 N$	$Nlog_2N$
OMP	$2K\log_2 N$	NK^3

V. Simulation and Result

The AT&T image library, which is the most well-known face recognition library, is used in this simulation. This library has 400 training images (40 subjects with each ten images). The pictures were taken at different lighting, times, facial expressions, and minor occlusion (glasses and no glasses). The people were photographed standing straight in the uniform background (with tolerance for some side movement) [18].

We tested the performance of the reconstruction algorithm: OMP, LASSO, and CVX by reducing the image dimensions by using the reduction factor (ρ) from 64 to 1024. The dimensionality reduction aims to have the condition of an underdetermined system and examine the effect of image size on the accuracy and computational time of three reconstruction algorithms. In this paper, we used the classical down-scaled method. The simulation results in term of accuracy and relative computation time as a function of dimensionality reduction is shown in Table II and Table III respectively. This results are also shown in Fig.2 and Fig.3 respectively.

TABLE II
REDUCTION FACTOR VS RECOGNITION RATE (%)

Reduction Factor (ρ)	**Recognition Rate (%)**		
	OMP	**LASSO**	**CVX**
64	78	79.5	79.5
128	99	92.5	92.5
256	78	94.5	94.5
512	58	89.5	89.5
1024	33.5	63	63

TABLE III
REDUCTION FACTOR VS COMPUTATION TIME (S)

Reduction Factor (ρ)	**Computation Time (s)**		
	OMP	**LASSO**	**CVX**
64	23	37	70
128	23	25	45
256	18	22	45
512	23	20	40
1024	19	20	38
Average	21.2	24.8	47.6

Considering the experimental results depicted in Table. II, the OMP algorithm achieve the highest recognition 99% accuracy on $\rho = 128$, and declined on higher reduction factor. On the other hand, LASSO and CVX algorithms have the same recognition rate trend, which gets a maximum of 94.5% accuracy at a reduction factor of 256. Table III shows that the OMP algorithm has the fastest computation compared to LASSO, with an average time of 21.2 seconds, which is about 12% faster. Although LASSO and CVX give the same accuracy, LASSO is twice faster than CVX. Fig. III shows LASSO and CVX algorithm tends to be more stable in accuracy than OMP. The selection of the reconstruction algorithm can affect the accuracy and computation time.

VI. CONCLUSION

An observation made in this study is that a signal can be rebuilt by making use of a reconstruction method that is founded on optimization research. The amount of time required to complete computations is another metric that may be used to evaluate the effectiveness of a particular algorithm. The computational time of the OMP algorithm is lower than the other sparse representation with ℓ_1-norm minimization algorithms, as we expect from the Greedy Algorithm. Sparse representations with ℓ_1-norm minimization algorithms always solve the problem by going through steps repeatedly. The OMP algorithms use the fast and efficient least-squares method, meaning they take much less time to run than other sparse representation algorithms that use the ℓ_1-norm. On the recognition stability, however, ℓ_1-norm algorithms such as LASSO and CVX produce a stable result over a wide range of compression factors.

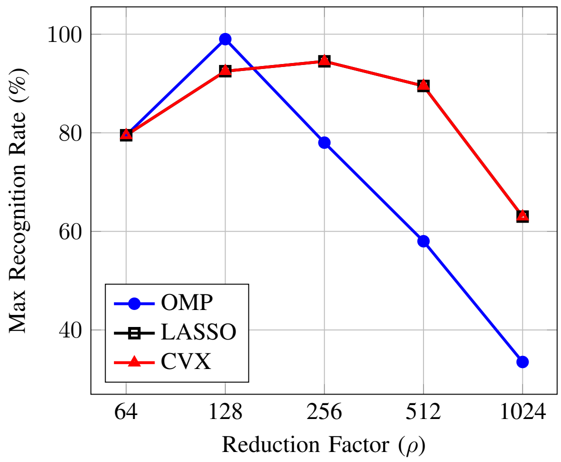

Fig. 2. Recognition Rate on OMP, LASSO and CVX Algorithm

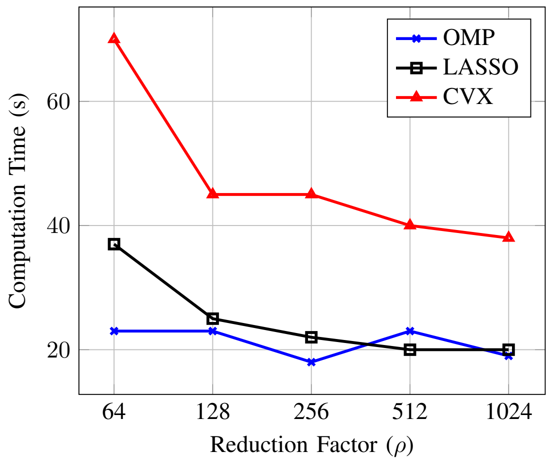

Fig. 3. Computation Time on OMP, LASSO, and CVX Algorithm

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