# bu nia

*By* Suci Aulia

# Comparison of Reconstruction Algorithm on Sparse Representation based Classification (SRC) for Face Recognition


1st Susmini Indriani Lestariningati
*School of Electrical Engineering and Informatics*
*Institut Teknologi Bandung*
Bandung, Indonesia
lestariningati@gmail.com

2nd Andriyan Bayu Suksmono
*School of Electrical Engineering and Informatics*
*Institut Teknologi Bandung*
Bandung, Indonesia
absuksmono@gmail.com

3rd Korediantо Usman
*School of Electrical Engineering, Telecommunication Eng. Dept.*
*Telkom University*
Bandung, Indonesia
korediantousman@telkomuniversity.ac.id

4th Ian Joseph Matheus Edward
*School of Electrical Engineering and Informatics*
*Institut Teknologi Bandung*
Bandung, Indonesia
ian@stei.itb.ac.id

5th Dewi Iswaratika
*School of Electrical Engineering, Telecommunication Eng. Dept.*
*Telkom University*
Bandung, Indonesia
dewiiswaratika@student.telkomuniversity.ac.id



***Abstract*—Sparse representation has gained the attention of pattern recognition and computer vision researchers, especially researchers working on face recognition. Many different algorithms have been proposed for sparse representation. It is necessary to find a solution to an optimization problem to recover $\mathbf{x}$ from the equation $\mathbf{y} = \mathbf{Ax}$. Only a few studies reported the reconstruction of the signals on SRC's algorithm. Therefore, this paper studies the comparison of OMP, LASSO, and CVX to help the readers understand the reconstruction algorithm's effect on SRC. The simulation result is that LASSO and CVX algorithms have the same recognition rate, but LASSO can compute twice faster than CVX. On the other hand, the OMP algorithm can give the highest recognition rate on a specific dimension of the image with a faster computation time than LASSO.**



***Index Terms*—Sparse Representation, Compressive Sensing, Reconstruction**


## I. Introduction

In signal processing and optimization, Compressive Sensing (CS) has been one of the most interesting topics over the past few decades. This theory was implemented for the first time by Donoho in 2006 [1] and popularized by Candés *et al.* in 2008 [2]. CS revolutionary transformed the paradigm for sensing or sampling, which challenges traditional signal acquisition techniques that use Shannon Theory [3], can take advantage of the fact that many natural signals are either sparse or compressible by choosing the appropriate basis [4].

Numerous aspects of signal processing, require the solution of a sparse approximation problem such as denoising [5], image-inpainting [6], target detection [7], computer vision [8] and pattern recognition [9], etc. Sparse representation refers to solving the system of equations $\mathbf{y} = \mathbf{Ax}$ when the matrix $\mathbf{A}$ has more columns than rows, and the vector $\mathbf{x}$ is sparse. They must recover a sparse signal from a collection of undersampled measurements. There are many sparse recovery algorithms have been proposed. Non-convex optimization techniques, convex relaxations, and greedy algorithms are the most common types of sparse recovery algorithm [10]. Fig. 1 depicted the classification of the sparse recovery algorithm based on these categories.

Two popular techniques for enforcing sparsity in the solution are the $\ell_0$-quasinorm (number of nonzero elements in the vector), which leads to an impossibly difficult numerical issue, and the $\ell_1$-norm. It is generally known that recovering vectors with more nonzero coefficients than the $\ell_1$-norm requires a regularization term like the $\ell_0$-quasinorm. The $\ell_1$-min problem can be resolved using standard convex optimization techniques. [11].

In computer vision, pattern recognition, and image analysis, finding sparse solutions has become an increasingly important technique. In particular, in the context of Face Recognition (FR), where the primary objective is to determine a person's identity based on an image of their face given a collection of example faces. The sparse representation-based classification (SRC) suggested by Wright *et al.*. provides a robust answer for FR problems. Such as dimensionality reduction using the downscale technique, handling occlusion, and image corruption [12].

The SRC technique is based on the fundamental idea that other examples of the same class can linearly represent an image of a face. Linearly, each class is distinct from the others. A face data set is a collection of images of people's faces that are organized into a matrix with the notation $\mathbf{A} \in \mathbf{R}^{w \times h}$ as a representation of the data training samples. In numerous image processing techniques, the vector representation version of the matrix $\mathbf{A}$ is denoted by $\mathbf{v} \in \mathbf{R}^m$, and $m = w \times h$, where $w$ and $h$ respectively represent the width and height of the face image. The accuracy in FR problems is determined by calculating this $\mathbf{x}$ value. The desired solution $\mathbf{x}$ is as sparse as possible. The majority of SRC's modification algorithms employ $\ell_1$-norm minimization. However, to classify the test image more accurately, we must use the most sparse value of $\mathbf{x}$.

In this paper, we study and simulated the reconstruction algorithm based on the $\ell_1$-norm minimization using convex optimization and $\ell_0$-norm minimization using the greedy algorithm. We compared these algorithm and saw the performance based on the accuracy and computation time. As far as we know, the SRC method's comparison of $\ell_0$-norm and $\ell_1$-norm reconstruction is not yet available.

- Sparse Recovery Algorithms
  - Convex Relaxation
    - Basis Pursuit (BP)
    - Least Absolute Shrinkage & Selection Operator (LASSO)
    - Least Angle Regression (LARS)
    - Approximate Message Passing (AMP)
    - Gradient Descent with Sparsification (GraDes)
    - Iterative Soft Thresholding (IST)
  - Non-Convex Optimization
    - Bayesian Compressive Sensing (BCS)
    - Focal Underdetermined System Solution (FOCUSS)
    - Iterative Reweighted Least Squares (IRLS)
  - Greedy Algorithm
    - Matching Pursuit (MP)
    - Matching Pursuit based on Least Squares (MPLS)
    - Orthogonal Matching Pursuit (OMP)
    - Subspace Pursuit (SP)
    - Stagewise Orthogonal Matching Pursuit (SOMP)
    - Compressive Sampling Matching Pursuit (CoSaMP)
    - Regulatized OMP (ROMP)
    - Generalized Orghogonal Matching Pursuit (GOAMP)
    - Gradient Pursuit (GP)
    - Multipath Matching Pursuit (MMP)

Fig. 1. Classification of Sparse Recovery Algorithm Adopted From [10]

## II. Mathematical Optimization

SRC recognition method belongs to mathematical optimization problem. The form of a mathematical optimization problem, or optimization problem, is given by [13]:

$$\begin{aligned} &\text{minimize} \quad f_0(x) \\ &\text{subject to} \quad f_i(x) \leq b_i, i = 1, ..., m \end{aligned} \tag{1}$$

where $x = (x_1, x_2, ..., x_n)$ is the optimization variables, $f_0 : \mathbf{R}^n \rightarrow \mathbf{R}$ is the objective function and $f_i : \mathbf{R}^n \rightarrow \mathbf{R}, i = 1, 2, ...m$ is the constraint functions, and the constants $b_1, ..., b_m$ are the limits, or bounds, for the constraints. The optimal solution of $\hat{x}$ has smallest value of $f_0$ among all vectors that satisfy the constraints. The optimization problems, in general, are generally difficult to solve, and many of the proposed solutions come with undesirable trade-offs—for example, extremely lengthy calculation times or an inability to reliably locate the optimal answer. However, there are some problem classes that can be handled in an effective manner and with a high degree of reliability by employing techniques such as least-squares, linear programming, and convex optimization.

### A. Least-Squares

The aim of a problem known as the least-squares problem is the sum of squares of terms represented by the form $a_i^T x - b_i$ [13]. This type of optimization issue does not involve any constraints.

$$\text{minimize} ||Ax - b||_2^2 = \sum_{i=1}^{k} (a_i^T x - b_i)^2 \tag{2}$$

where $A \in \mathbf{R}^{m \times n}$ (with $m \geq n$), $a_i^T$ are the rows of $A$, and the vector $x \in \mathbb{R}^n$ is the optimization variable. The solution of problem (2):

$$\begin{aligned} (A^T A)x &= A^T b \\ x &= (A^T A)^{-1} A^T b \end{aligned} \tag{3}$$

In a time approximately proportional to $n^2 k$, the least-squares problem can be solved.

### B. Linear Programming

Linear programming is another major sub-field of optimization problems; in this type of issue, both the constraint functions and the objective functions are linear.

$$\begin{aligned} &\text{minimize} \quad c^T x \\ &\text{subject to} \quad a_x^T \leq b_i, i = 1, ..., m \end{aligned} \tag{4}$$

where the vectors $x, a_1, ..., a_m \in \mathbf{R}^n$ and scalars $b_1, ..., b_m \in \mathbf{R}$ are problem parameters that specify the objective and constraint functions respectively. It is not possible to solve a linear program using a straightforward analytical method in the same way that a least-squares problem may be solved. However, there are many very effective ways to solve linear programs, such as Dantzig's simplex method.

### C. Convex Optimization

One of the following formulations can represent a convex optimization problem [13]:

$$\begin{aligned} &\text{minimize} \quad f_0(x) \\ &\text{subject to} \quad f_i(x) \leq b_i, i = 1, ..., m \end{aligned} \tag{5}$$

where the functions $f_0, ..., f_m : \mathbf{R}^n \rightarrow \mathbf{R}$ are convex, satisfy:

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y) \tag{6}$$

for all $x, y \in \mathbf{R}^n$ and all $\alpha, \beta \in \mathbf{R}$ with $\alpha + \beta = 1, \alpha \geq 0, \beta \geq 0$

## III. Sparse Representation based Classification

The robust FR using sparse representation was written by Wright *et al.* as follows [12]:

$$\mathbf{y} = \mathbf{A}\mathbf{x} \tag{7}$$

In the case of classification on face recognition, $\mathbf{y} \in \mathbf{R}^{M\times1}$ represented the image that needed to be identified, $\mathbf{A} \in \mathbf{R}^{M\times N}$ is training sample column-wise database matrix and $\mathbf{x} \in \mathbf{R}^{N\times1}$ is a sparse matrix. If only $K(K \ll N)$ elements of $\mathbf{x}$ are non zero and the rest elements in $\mathbf{x}$ are zero, we call the signal $\mathbf{y}$ is $K$-sparse. We want to find $\mathbf{x}$ if $\mathbf{y}$ and $\mathbf{A}$ are known. The sparsest solution $\hat{\mathbf{x}}$ is, there fore given by problem (8):

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_0 \quad \text{s.t} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \tag{8}$$

Despite the fact that this problem is NP-hard, it is possible to solve it using greedy methods under certain conditions (depending on the values of $N$, $M$, $K$, and $\mathbf{A}$) [14].

Recent studies have demonstrated that if a representative solution is produced by applying the $\ell_1$-norm minimization with adequate sparsity, then the solution can be equal to the solution obtained by $\ell_0$-norm with a high probability [15]. In addition, the problem of $\ell_1$-norm optimization has an analytical solution and can be handled in polynomial time. As a result, extensive sparse representation approaches that incorporate $\ell_1$-norm minimization have been presented to enhance the theory of sparse representation. The most common and popular structures of sparse representation with the $\ell_1$-norm minimization, which are very similar to sparse representation with the $\ell_0$-norm minimization, are typically employed to solve the following problems:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_1 \quad \text{s.t} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \tag{9}$$

In the case of noisy measurement, the optimization problem given in problem (9) is relaxed to the following problem [15]:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^N}{\arg\min} ||\mathbf{x}||_1 \quad \text{s.t} \quad ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 \leq \varepsilon \tag{10}$$

The test image $\mathbf{y}$ is classify based on the approximation by assigning it to the class of object with minimum residual between $\mathbf{y}$ and $\hat{\mathbf{y}}$:

$$\min_i r_i(\mathbf{y}) = ||\mathbf{y} - A\delta_i(\hat{\mathbf{x}})||_2 \tag{11}$$

Wright *et al.* studied the implication of feature extraction, which carries over the SRC framework, by reducing the dimensionality of data and computational cost [12]. One key element in the practical application of SRC is the dimension reduction of the training samples. The number of calculations directly influences the initial size of the training samples, affecting the algorithm's complexity. This dimension reduction is also perceived as feature extraction from original sample images. A formula for reduction factor ($\rho$) from the raw image's $\mathbb{R}^M$ to a lower-dimensional feature $\mathbb{R}^d$ ($d << M$) as follows:

$$\rho = \frac{\text{M (raw image)}}{\text{d (reduced image)}} \tag{12}$$

## IV. Greedy Approach

The task of addressing sparse representation with $\ell_0$-norm regularization, often known as the problem (8), is considered an NP-hard problem. The greedy strategy offers a method for obtaining an approximate solution for sparse representation problems. In reality, the greedy approach cannot directly resolve the optimization problem; all it can do is search for an approximation of a solution to a problem (8). Greedy algorithms use the notion of dictionary atomic matching to seek the optimal global solution from the local optimal. This algorithm is utilized by gradually raising the practical column to achieve the closest answer to the initial signal. The initial practical set is the empty set, which is updated column by column by locating the minimum reconstruction residual; the operation concludes when this residual is smaller than a threshold value.

Two examples of typical sparse representation greedy algorithms are The matching pursuit (MP) and the orthogonal matching pursuit (OMP) [16]. Both MP and OMP have speedy recovery times and inexpensive implementation costs. The following table compares the characteristics of the two greedy algorithms [17].

TABLE I
Comparison of Greedy Algorithms [16]

| Algorithm Name | Samples of Observations | Complexity |
|---|---|---|
| MP | $K\log_2 N$ | $Nlog_2N$ |
| OMP | $2K\log_2 N$ | $NK^3$ |

## V. Simulation and Result

The AT&T image library, which is the most well-known face recognition library, is used in this simulation. This library has 400 training images (40 subjects with each ten images). The pictures were taken at different lighting, times, facial expressions, and minor occlusion (glasses and no glasses). The people were photographed standing straight in the uniform background (with tolerance for some side movement) [18].

We tested the performance of the reconstruction algorithm: OMP, LASSO, and CVX by reducing the image dimensions by using the reduction factor ($\rho$) from 64 to 1024. The dimensionality reduction aims to have the condition of an underdetermined system and examine the effect of image size on the accuracy and computational time of three reconstruction algorithms. In this paper, we used the classical down-scaled method. The simulation results in term of accuracy and relative computation time as a function of dimensionality reduction is shown in Table II and Table III respectively. This results are also shown in Fig.2 and Fig.3 respectively.

TABLE II
REDUCTION FACTOR VS RECOGNITION RATE (%)

| Reduction Factor ($\rho$) | Recognition Rate (%) | | |
|---|---|---|---|
| | OMP | LASSO | CVX |
| 64 | 78 | 79.5 | 79.5 |
| 128 | 99 | 92.5 | 92.5 |
| 256 | 78 | 94.5 | 94.5 |
| 512 | 58 | 89.5 | 89.5 |
| 1024 | 33.5 | 63 | 63 |

TABLE III
REDUCTION FACTOR VS COMPUTATION TIME (S)

| Reduction Factor ($\rho$) | Computation Time (s) | | |
|---|---|---|---|
| | OMP | LASSO | CVX |
| 64 | 23 | 37 | 70 |
| 128 | 23 | 25 | 45 |
| 256 | 18 | 22 | 45 |
| 512 | 23 | 20 | 40 |
| 1024 | 19 | 20 | 38 |
| **Average** | 21.2 | 24.8 | 47.6 |

Considering the experimental results depicted in Table. II, the OMP algorithm achieve the highest recognition 99% accuracy on $\rho = 128$, and declined on higher reduction factor. On the other hand, LASSO and CVX algorithms have the same recognition rate trend, which gets a maximum of 92.5% accuracy at a reduction factor of 256. Table III shows that the OMP algorithm has the fastest computation compared to LASSO, with an average time of 21.2 seconds, which is about 12% faster. Although LASSO and CVX give the same accuracy, LASSO is twice faster than CVX. Fig. III shows LASSO and CVX algorithm tends to be more stable in accuracy than OMP. The selection of the reconstruction algorithm can affect the accuracy and computation time.

## VI. CONCLUSION

This paper observes that a signal has been reconstructed using a reconstruction algorithm based on optimization studies. The amount of time required to complete computations is another metric that may be used to evaluate the effectiveness of a particular algorithm. The computational time of the OMP algorithm is lower than the other sparse representation with $\ell_1$-norm minimization algorithms, as we expect from the Greedy Algorithm. Sparse representations with $\ell_1$-norm minimization algorithms always solve the problem by going through steps repeatedly. The OMP algorithms use the fast and efficient least-squares method, meaning they take much less time to run than other sparse representation algorithms that use the $\ell_1$-norm. On the recognition stability, however, $\ell_1$-norm algorithms such as LASSO and CVX produce a stable result over a wide range of compression factors.

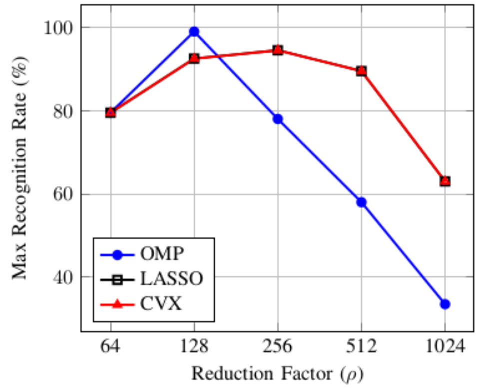


Fig. 2. Recognition Rate on OMP, LASSO and CVX Algorithm

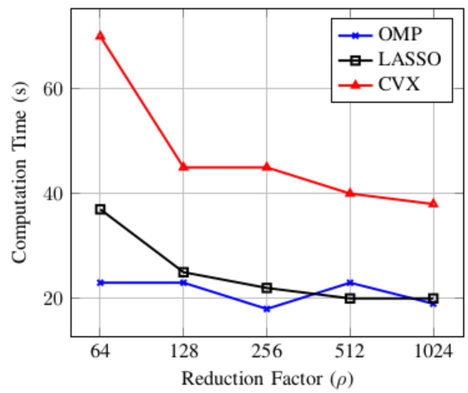


Fig. 3. Computation Time on OMP, LASSO, and CVX Algorithm

## REFERENCES

[1] D. L. Donoho, "Compressed Sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289 – 1306, Apr. 2006.
[2] E. Candes and M. Wakin, "An Introduction To Compressive Sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, Mar. 2008. [Online]. Available: http://ieeexplore.ieee.org/document/4472240/
[3] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.
[4] E. J. Candés and M. B. Wakin, "An Introduction to Compressive Sampling," *IEEE Signal Processing Magazine*, Mar. 2008.
[5] A. Gholami and S. M. Hosseini, "A general framework for sparsity-based denoising and inversion," *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5202–5211, 2011.
[6] M. Fadili, J.-L. Starck, and F. Murtagh, "Inpainting and zooming using sparse representations," *The Computer Journal*, vol. 52, no. 1, pp. 64–79, 2009.
[7] L. Yao and X. Du, "Identification of underwater targets based on sparse representation," *IEEE Access*, vol. 8, pp. 215–228, 2020.
[8] Z. Wang, J. Yang, H. Zhang, Z. Wang, Y. Yang, D. Liu, and T. S. Huang, *Sparse Coding and Its Applications in Computer Vision*. USA: World Scientific Publishing Co., Inc., 2015.
[9] J. Wright, Y. Ma, J. Mairal, G. Sapiro, T. S. Huang, and S. Yan, "Sparse representation for computer vision and pattern recognition," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 1031–1044, 2010.

[10] E. Crespo Marques, N. Maciel, L. Naviner, H. Cai, and J. Yang, "A review of sparse recovery algorithms," *IEEE Access*, vol. 7, pp. 1300–1322, 2019.

[11] F. Keinert, D. Lazzaro, and S. Morigi, "A robust group-sparse representation variational method with applications to face recognition," *IEEE Transactions on Image Processing*, vol. 28, no. 6, pp. 2785–2798, 2019.

[12] J. Wright, A. Y. Yang, A. Ganesh, S. S. Sastry, and Y. Ma, "Robust Face Recognition via Sparse Representation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, no. 2, pp. 210 – 227, Apr. 2008.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[14] J. Tropp, "Greed is good: algorithmic results for sparse approximation," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2231–2242, 2004.

[15] ——, "Just relax: convex programming methods for identifying sparse signals in noise," *IEEE Transactions on Information Theory*, vol. 52, no. 3, pp. 1030–1051, 2006.

[16] J. Dong and L. Wu, "Comparison and simulation study of the sparse representation matching pursuit algorithm and the orthogonal matching pursuit algorithm," in *2021 International Conference on Wireless Communications and Smart Grid (ICWCSG)*, 2021, pp. 317–320.

[17] R. Manchanda and K. Sharma, "A review of reconstruction algorithms in compressive sensing," in *2020 International Conference on Advances in Computing, Communication & Materials (ICACCM)*, 2020, pp. 322–325.

[18] "The ORL Dataset." [Online]. Available: https://cam-orl.co.uk/facedatabase.html

# bu nia

ORIGINALITY REPORT

18%

SIMILARITY INDEX

PRIMARY SOURCES

| | | |
|---|---|---|
| 1 | www.pdf-search-engine.com<br>Internet | 105 words — 4% |
| 2 | Junshuo Dong, Lingda Wu. "Comparison and Simulation Study of the Sparse Representation Matching Pursuit Algorithm and the Orthogonal Matching Pursuit Algorithm", 2021 International Conference on Wireless Communications and Smart Grid (ICWCSG), 2021<br>Crossref | 50 words — 2% |
| 3 | hdl.handle.net<br>Internet | 50 words — 2% |
| 4 | arxiv.org<br>Internet | 47 words — 2% |
| 5 | Fritz Keinert, Damiana Lazzaro, Serena Morigi. "A Robust Group-Sparse Representation Variational Method With Applications to Face Recognition", IEEE Transactions on Image Processing, 2019<br>Crossref | 36 words — 1% |
| 6 | Zheng Zhang, Yong Xu, Jian Yang, Xuelong Li, David Zhang. "A Survey of Sparse Representation: Algorithms and Applications", IEEE Access, 2015<br>Crossref | 26 words — 1% |

| | | |
|---|---|---|
| 7 | Manal El Tanab, Walaa Hamouda. "Resource Allocation for Underlay Cognitive Radio Networks: A Survey", IEEE Communications Surveys & Tutorials, 2017<br>Crossref | 24 words — 1% |
| 8 | www.freepatentsonline.com<br>Internet | 23 words — 1% |
| 9 | www.esat.kuleuven.be<br>Internet | 20 words — 1% |
| 10 | Imran Naseem. "Sparse Representation for Ear Biometrics", Lecture Notes in Computer Science, 2008<br>Crossref | 16 words — 1% |
| 11 | Rachit Manchanda, Kanika Sharma. "A Review of Reconstruction Algorithms in Compressive Sensing", 2020 International Conference on Advances in Computing, Communication & Materials (ICACCM), 2020<br>Crossref | 10 words — < 1% |
| 12 | vdoc.pub<br>Internet | 10 words — < 1% |
| 13 | Zhangyang Wang, Houqiang Li, Qing Ling, Weiping Li. "Robust Temporal-Spatial Decomposition and Its Applications in Video Processing", IEEE Transactions on Circuits and Systems for Video Technology, 2013<br>Crossref | 9 words — < 1% |
| 14 | kth.diva-portal.org<br>Internet | 9 words — < 1% |
| 15 | pt.scribd.com<br>Internet | 9 words — < 1% |

| | | |
|---|---|---|
| 16 | www.koreascience.or.kr<br>Internet | 9 words — < 1% |
| 17 | www.yongxu.org<br>Internet | 9 words — < 1% |

| | | | |
|---|---|---|---|
| EXCLUDE QUOTES | ON | EXCLUDE SOURCES | < 9 WORDS |
| EXCLUDE BIBLIOGRAPHY | ON | EXCLUDE MATCHES | < 9 WORDS |