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TEKNIK

PID CONTROL OF A THREE-DEGREES-OF-FREEDOM MODEL HELICOPTER

MUHAMMAD ARIA

Department of Electrical Engineering
Engineering and Computer Science Faculty
Universitas Komputer Indonesia

Helicopter dynamic are in general nonlinear, time-varying and may be highly uncertain. This paper presents the design and implementation of a Proportional-Integral-Derivative (PID) Controller to control the elevation and travel of three-degrees-of freedom (3DOF) Helicopter. The controller is linear time-invariant and can be realized easily. The simulation results show that the designed control system can guarantee high precision altitude and elevation control under multi-operation points.

Index Terms – Attitude control, 3DOF, PID, helicopter dynamic, MIMO systems

INTRODUCTION

In this paper, a Proportional-Integral-Derivative (PID) controller is applied to a laboratory helicopter model used in the experiment is a laboratory-scale three-degrees-of freedom (3DOF) helicopter produced by Quanser Consulting, Inc. The 3DOF helicopter control system is a nonlinear MIMO uncertain system with unknown constant parameters, bounded disturbance and nonlinear uncertainty. Our control goal is to have the attitude of the helicopter track a reference signal by output feedback.

The 3DOF helicopter consists of a base upon which a long arm is mounted. The arm carries the “helicopter body” on one end and a counterweight on the other. The arm can tilt about an “elevation” axis as well as swivel about a vertical (travel) axis. Quadrature optical encoders mounted on these axes allow for measuring the elevation and travel of the arm. The pitch angle is measured via a third encoder. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to the motors. The force generated by the propellers causes the helicopter body to lift off the ground. The purpose of the counterweight is to reduce the power requirements on the motors. Electrical signals to and from the arm and helicopter are channeled through the slipring to eliminate tangled wires, reduce friction and allow for unlimited and unhindered travel.

The purpose of the experiment is to design a PID controller to maneuver the helicopter body to track and regulate the elevation and travel of the 3DOF Helicopter.

The paper is organized as follows. Dynamic model of a helicopter is provided in Section II. Designing the control system is presented in Section III. The simulation results are provided in Section IV. In Section V, we conclude with conclusion.

DYNAMIC MODEL

The research presented in this paper is based on a mathematical model of a 3-DOF laboratory helicopter system from Quanser Consulting, Inc. The 3-DOF helicopter consists of a base upon which an arm is mounted. The arm carries the helicopter body on one end and a counter weight on the other end. The arm can pitch about an elevation axis as well as swivel about a vertical axis. Encoders that are m

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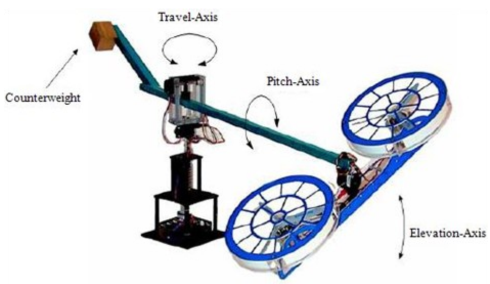

Figure 1. Laboratory Helicopter from Quanser Consulting, Inc

The system dynamics can be described by the following state model [1,3]:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where

$$x = [\varepsilon \quad p \quad \lambda \quad \dot{\varepsilon} \quad \dot{p} \quad \dot{\lambda} \quad \zeta \quad \gamma]^T \tag{3}$$

$$y = [\varepsilon \quad p \quad \lambda]^T \tag{4}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.15 & 0.15 \\ 1.02 & -0.12 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

$$u = \begin{bmatrix} V_f \\ V_b \end{bmatrix} \tag{9}$$

The symbols used in the above model are described in Table 1.

Table 1. Notation and units used in the laboratory helicopter model

Symbol	Unit	Description
ε	Degree	Elevation angle
p	Degree	Pitch angle
λ	Degree	Travel angle
V_f	Volt	Voltages applied to the front motor
V_b	Volt	Voltages applied to the back motor

The model has been implemented in LabVIEW. **Figure 2** shows Front Panel of LabVIEW program and **Figure 3** shows the Block Diagram. This model simulates system dynamics and measurement. It reduces the A and B matrices to a set of integrators and gains. The elevation integrator is limited to positive values in order to simulate the helicopter landing on the ground. Augmenting the motor voltages by the constant simulates gravitational bias. Multiplying by a calibration constant that converts radians to degrees simulates encoder measurements. Because high-pass filters function as differentiators at frequencies below the passband, they are used to differentiate the three displacement states.

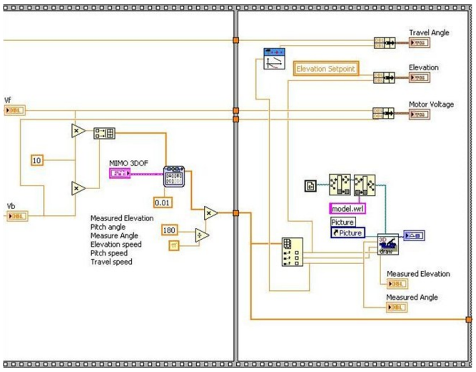

Figure 3. Block Diagram of 3 DOF Helicopter Simulation in LabVIEW

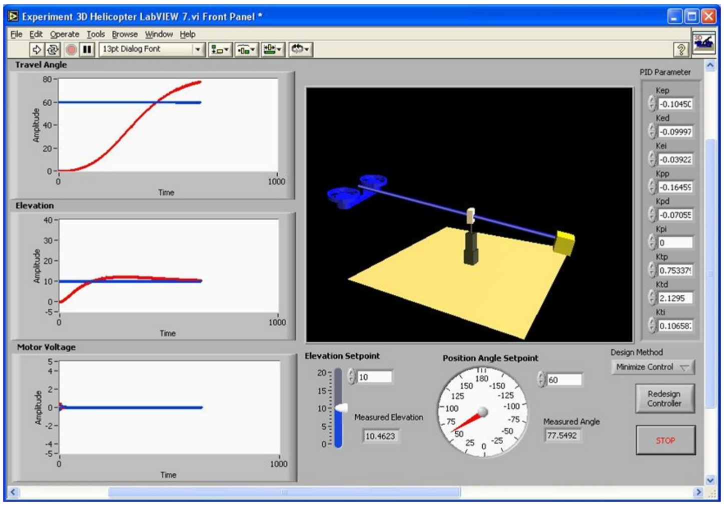

Figure 2. Front Panel of 3 DOF Helicopter Simulation in LabVIEW

DESIGNING THE CONTROL SYSTEM

In this section a linear Proportional-Integral-Derivative (PID) controller is designed to regulate the elevation and travel angles of the 3-DOF Helicopter to desired positions. The PID control gains are computed using the Linear-Quadratic Regular algorithm.
Using the weighting matrices

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} \qquad (10)$$

and

$$R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} \qquad (11)$$

and the state-space matrices (A,B) found previously, the control gain

$$K = \begin{bmatrix} 37.67 & 37.67 \\ 13.21 & -13.21 \\ -11.50 & 11.50 \\ 20.95 & 20.95 \\ 4.769 & -4.769 \\ -16.10 & 16.10 \\ 10.00 & 10.00 \\ -1.00 & 1.00 \end{bmatrix}^T \qquad (12)$$

is calculated by minimizing the cost function

$$J = \int_0^\infty x_i^T Q x_i + u^T R u \, dt \qquad (13)$$

with the Matlab LQR command. Examining the feedback gains obtained from LQR design, we note the following feature : the second row gains have the exact same magnitudes as the first row.

$$K = \begin{bmatrix} k_{11} & k_{11} \\ k_{12} & -k_{12} \\ -k_{13} & k_{13} \\ k_{14} & k_{14} \\ k_{15} & -k_{15} \\ -k_{16} & k_{16} \\ k_{17} & k_{17} \\ -k_{18} & k_{18} \end{bmatrix}^T \qquad (14)$$

Further examination reveals that the sum of the rows results in :

$$V_f + V_b = V_s = -[2k_{11}(\varepsilon - \varepsilon_c) + 2k_{14}\dot{\varepsilon} + 2k_{17}\zeta] \qquad (15)$$

which can be re-written as

$$V_s = k_{ep}(\varepsilon - \varepsilon_c) + k_{ed}\dot{\varepsilon} + k_{ei}\int(\varepsilon - \varepsilon_c) \qquad (16)$$

which is the Proportional+Integral+ Derivative (PID) controller about the elevation axis.

Examining the difference between the gains, the feedback loop can be decomposed into two loops : one for pitch and one for travel.

$$V_f - V_b = V_d = -[2k_{12}p - 2k_{15}\dot{p}] - [2k_{13}(\lambda - \lambda_c) + 2k_{16}\dot{\lambda} + 2k_{18}\zeta] \qquad (17)$$

The above equation can be re-written as.

$$V_d = -2k_{12}[p - p_c] - 2k_{15}\dot{p} \qquad (18)$$

which is a PD loop to command the pitch to track the desired pitch. The desired pitch is defined as.

$$p_c = -\frac{[2k_{13}(\lambda - \lambda_c) + 2k_{16}\dot{\lambda} + 2k_{18}\zeta]}{2k_{12}} \qquad (19)$$

which is another PID loop that controls the travel position. We now have the following control equations.

$$p_c = k_{Ip}(\lambda - \lambda_c) + k_{Id}\dot{\lambda} + k_{Ii}\zeta \qquad (20)$$

$$V_s = k_{ep}(\varepsilon - \varepsilon_c) + k_{ed}\dot{\varepsilon} + k_{ei}\int(\varepsilon - \varepsilon_c) \quad (21)$$

$$V_d = k_{pp}[p - p_c] - k_{pd}\dot{p} \quad (22)$$

And we obtain

$$V_f = 0.5\,(V_s + V_d) \quad (23)$$

$$V_b = 0.5\,(V_s - V_d) \quad (24)$$

The implementation of the controller is shown in figure 4 – 7. It consists of three loops. The pitch command is limited to +/- 90 degrees.

SIMULATION RESULTS

In this simulation, the reference signals for the elevation and travel angles are changes between 100º to 60º and between 20º to 10º to simulate the demands given by the pilot as shown in **Figure 8** and **Figure 9**. Also, the sampling time and simulation time used are 0.1 and 270 seconds, respectively.

Figure 10 and **Figure 11** shows the transient response and steady-state response of the elevation angle and travel angle to step signal. From **Figure 10** and **Figure 11**, one sees that the percent overshoots of elevation and travel angles are approximate to 22.5 % and 40 % respectively. The settling time of elevation and travel angles are approximate to 10s and 20s with 5% criterion. And the steady state error of the elevation and travel angles are approximate to zero.

The corresponding voltage signals applied to the rotors are shown in **Figure 12** and **Figure 13**.

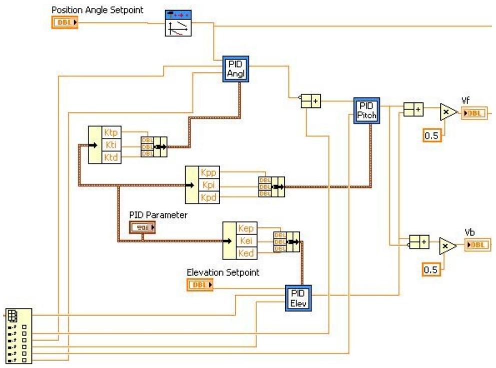

Figure 4. LabVIEW Feedback Controlled Block Diagram

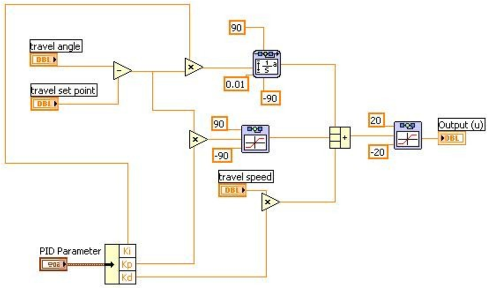

Figure 5. PID Angle Controller

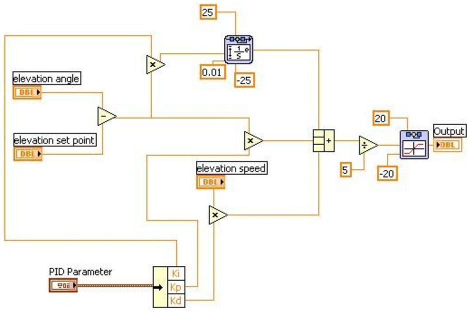

Figure 7. PID Elevation Controller

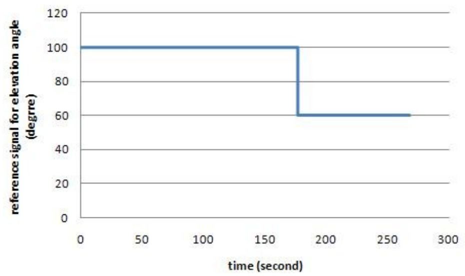

Figure 8. Reference signal for the elevation angle

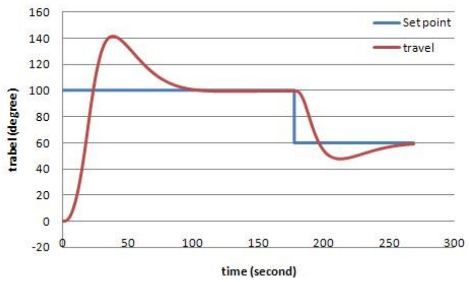

Figure 11. Control simulation results for the travel angle

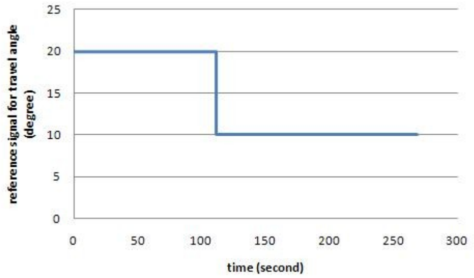

Figure 9. Reference signal for the travel angle

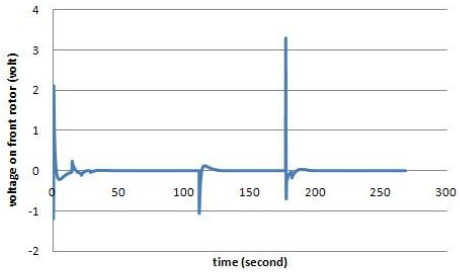

Figure 12. Voltage applied on the front motor

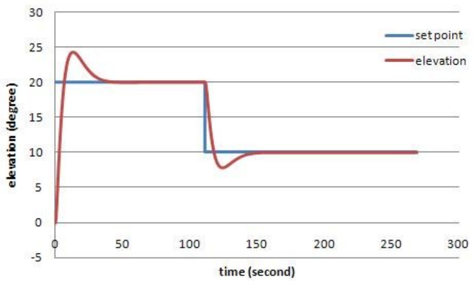

Figure 10. Control simulation results for the elevation angle

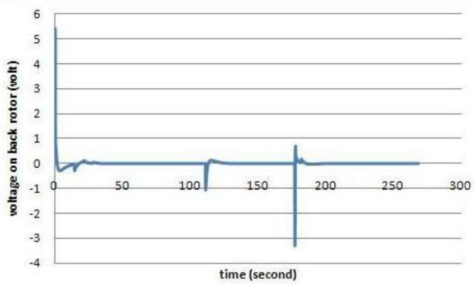

Figure 13. Voltage applied on the back rotor

It can be seen from the **Figures 10** and **Figure 11** that when the helicopter is operating in a wide range of its flight envelope, the tracking performance looks acceptable; however, large overshoot is not avoidable.. This is shown in Tables 3 which show the percent overshoot and settling time for the control algorithms

Table 3. Overshoot and Settling time for elevation and travel control

	Percentage Overshoot	Settling Time (seconds)
Elevation	22.5 %	40
Travel	40 %	100

CONCLUSIONS

In this paper, we have presents a LQR control the elevation and travel of a laboratory helicopter system. The performance of the algorithm is illustrated and the results show a good performance in term of tracking error, overshoot and settling time.

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