IDENTITAS PARSEVAL

Identitas ini menyatakan bahwa

$$\frac{1}{L}\int_{-L}^{L}\left(f(x)\right)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

Jika a_n dan b_n adalah koefisien Fourier yang bersesuaian dengan $f(x)$ dan jika $f(x)$ memenuhi syarat dirichlet

Contoh:

Tentukan jumlah dari deret $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots+\frac{1}{n^4}+\cdots$ jika diketahui koefisien deret Fourier untuk $f(x) = x, 0 < x < 2$, $a_0 = 2, a_n = \frac{4}{n^2\pi^2}(\cos n\pi - 1), b_n = 0$ dan $L = 2$

Jawaban:

Kesamaan Parseval:

$$\frac{1}{2}\int_{-2}^{2}(x)^2 dx = \frac{2^2}{2} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2\pi^2}(\cos n\pi - 1)\right)^2$$

$$\frac{1}{2}\left[\frac{1}{3}x^3 \,\Big|_{-2}^{2} = 2 + \sum_{n=1}^{\infty}\frac{16}{n^4\pi^4}(\cos n\pi - 1)^2\right.$$

$$\frac{8}{3} = 2 + \frac{16}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}(-2)^2$$

$$\frac{2}{3} = +\frac{64}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} \leftrightarrow \frac{\pi^4}{96} = \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots+\frac{1}{(2n-1)^4}+\cdots$$

Maka

$$\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots+\frac{1}{n^4}+\cdots = \left(\frac{1}{1^4}+\frac{1}{3^4}+\cdots+\frac{1}{(2n-1)^4}+\cdots\right)+\left(\frac{1}{2^4}+\frac{1}{4^4}+\cdots+\frac{1}{(2n)^4}+\cdots\right)$$

$$= \frac{\pi^4}{96}+\left(\frac{1}{2^4}+\frac{1}{4^4}+\cdots+\frac{1}{(2n)^4}+\cdots\right) = \frac{\pi^4}{96}+\frac{1}{2^4}\left(\frac{1}{1^4}+\frac{1}{2^4}+\cdots+\frac{1}{(n)^4}+\cdots\right)$$

$$\left(1-\frac{1}{2^4}\right)\left(\frac{1}{1^4}+\frac{1}{2^4}+\cdots+\frac{1}{n^4}+\cdots\right) = \frac{\pi^4}{96} \leftrightarrow \frac{15}{16}\left(\frac{1}{1^4}+\frac{1}{2^4}+\cdots+\frac{1}{n^4}+\cdots\right) = \frac{\pi^4}{96}$$

$$\left(\frac{1}{1^4}+\frac{1}{2^4}+\cdots+\frac{1}{n^4}+\cdots\right) = \frac{\pi^4}{90}$$