Bab 2: Sistem Diskrit

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Notasi Sinyal Diskrit

- Simbol sinyal diskrit : x(n) atau x(k); y(n) atau y(k), h(n) atau h(k)
- Pergeseran:
  x(n-1) menyatakan sampel ke (n-1)T, dengan T adalah perioda sampling
- **Deret diskrit Impuls**

$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n-k)$$

Unit impulse : $\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

Unit step

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Format Sinyal Diskrit

- Dalam bentuk fungsi

$$x(n) = \begin{cases} 1 & n = integer\ ganjil \\ 0 & n = interger\ genap \end{cases}$$

- Dalam bentuk tabel

n	… -2 -1 0 1 2 3 4 …
X(n)	… 0 1 2 3 2 1 1 ..

- Dalam bentuk deret
  x(n) = {0 0 0 1 2 3 4 5 6}
  ↑

  x(n-2) = {0 0 0 1 2 3 4 5 6}
  ↑

Sinyal Sinus

Bentuk analog sinyal sinus :

$$x(t) = A\sin(\Omega t + \phi) = A\sin(2\pi f t + \phi), \quad \Omega = 2\pi f$$

Bentuk digital sinyal sinus :

$$x(n) = A\sin(\Omega n T + \phi) = A\sin(2\pi f n T + \phi)$$
$$= A\sin(\omega n + \phi) = A\sin(\pi F n + \phi),$$

Frekuensi Digital : $\omega = \Omega T = \frac{2\pi f}{f_s}$

Frekuensi Digital ternormalisasi : $F = \frac{\omega}{\pi} = \frac{f}{(f_s/2)}$

Table 3.1 Units, relationships, and ranges of four frequency variables

Variables	Unit	Relationship	Range
Ω	Radians per second	$\Omega = 2\pi f$	$-\infty < \Omega < \infty$
f	Cycles per second (Hz)	$f = \frac{F f_s}{2} = \frac{\omega}{2\pi T}$	$-\infty < f < \infty$
ω	Radians per sample	$\omega = \Omega T = \pi F$	$-\pi \leq \omega \leq \pi$
F	Cycles per sample	$F = \frac{f}{(f_s/2)}$	$-1 \leq F \leq 1$

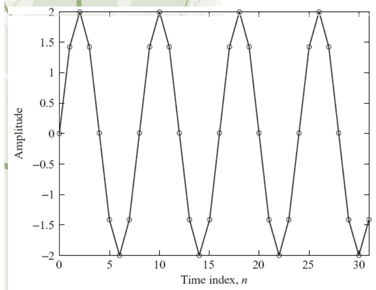
Contoh : Memperoleh 32 data sampling dari sinyal analog sinus

Bangkitkan 32 sample dari gelombang sinus dengan A = 2, f = 1000 Hz
fs = 8 kHz menggunakan Matlab

$$F = \frac{f}{(f_s/2)} = 0.25 \quad \omega = \pi F = 0.25\pi$$

Menggunakan MATLAB diperoleh:

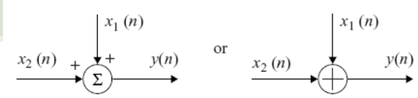
$$x(n) = 2\sin(\omega n),\ n = 0, 1, \ldots, 31$$



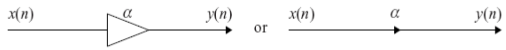

```
n = [0:31]                  ; % Time index n
omega = 0.25*pi             ; % Digital frequency
xn = 2*sin(omega*n)         ; % Sinewave generation
plot(n, xn, '-o')           ; % Samples are marked by
                                'o'

xlabel('Time index, n');
ylabel('Amplitude');
axis([0 31 -2 2]);
save sine.dat xn -ascii ;
```


Operasi Matematik Sinyal Diskrit

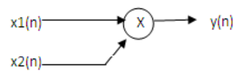
- **Penjumlahan :**
  y(n) = x1(n) + x2(n)



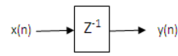
- **Amplitudo scaling** :
  y(n) = ax(n)



- **Perkalian** :
  y(n) = x1(n) . x2(n)



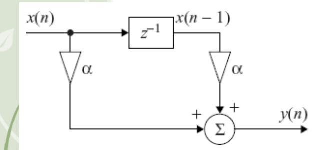
- **Delay negatif dan delay positif** :
  y(n) = x(n-d); y(n) = x(n+d)



- **Konvolusi diskrit :**

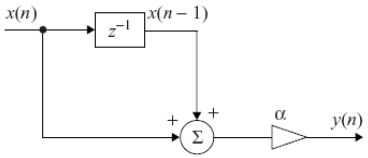
$$y(n) = x(n) * h(n) = h(n) * x(n) = \sum_{l=-\infty}^{\infty} x(l)h(n-l) = \sum_{l=-\infty}^{\infty} h(l)x(n-l).$$

Contoh : Gambarkan arsitektur $y(n) = \alpha x(n) + \alpha x(n-1).$




```
amov  #alpha,XAR1        ; AR1 points to α
amov  #temp,XAR2         ; AR2 points to temp
amov  #yn,XAR4           ; AR4 points to yn
mov  *(x1n),AC0          ; AC0 = x1(n)
add  *(x2n),AC0          ; AC0 = x1(n)+x2(n)
mov  AC0,*AR2            ; temp = x1(n)+x2(n),
                           pointed by AR2
mpy *AR1,*AR2,AC1        ; AC1 = α *[x1(n)+x2(n)]
mov AC1,*AR4             ; yn = α *[x1(n)+x2(n)]
```

Lebih efektif jika : $y(n) = \alpha\,[x(n) + x(n-1)]$

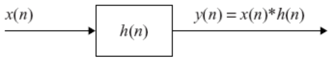


Sifat Sistem

- Linear Time Invariant:
  memiliki sifat superposisi dan tiap koefisien konstan
- Kasualitas:
  Output hanya bergantung dari input sekarang dan kondisi akan datang

Konsep Sistem Linear Time Invariant

- h(n) adalah respon impulse, output dari sistem jika inputnya impulse



Contoh tentukan koefisien respon impulse dari :

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2).$$

Jawab:

$$\begin{aligned} h(0) &= y(0) = b_0 \cdot 1 + b_1 \cdot 0 + b_2 \cdot 0 = b_0 \\ h(1) &= y(1) = b_0 \cdot 0 + b_1 \cdot 1 + b_2 \cdot 0 = b_1 \\ h(2) &= y(2) = b_0 \cdot 0 + b_1 \cdot 0 + b_2 \cdot 1 = b_2 \\ h(3) &= y(3) = b_0 \cdot 0 + b_1 \cdot 0 + b_2 \cdot 0 = 0 \\ &\vdots \end{aligned}$$

h(n) = $\{b_0, b_1, b_2, 0, 0, \ldots\}$.

Filter

- FIR (Finite Impulse Response) / Recursive

$$\begin{aligned} y(n) &= b_0 x(n) + b_1 x(n-1) + \cdots + b_{L-1} x(n-L+1) \\ &= \sum_{l=0}^{L-1} b_l x(n-l) \end{aligned}$$

- IIR (Infinite Impulse Response) / Non-recursive

$$\begin{aligned} y(n) &= b_0 x(n) + b_1 x(n-1) + \cdots + b_{L-1} x(n-L+1) - a_1 y(n-1) - \cdots - a_M y(n-M) \\ &= \sum_{l=0}^{L-1} b_l x(n-l) - \sum_{m=1}^{M} a_m y(n-m). \end{aligned}$$

Fungsi Matlab:

FIR:

```
yn = filter(b, 1, xn);
```

IIR:

```
yn = filter(b, a, xn);
```


Contoh: Persamaan digital I/O dinyatakan dengan $y(n) = bx(n) - ay(n-1)$

- Jika $y(n) = 0 \text{ for } n < 0$ dan $x(n) = \delta(n)$
  Maka persamaan output IIRnya adalah

$$\begin{aligned} y(0) &= bx(0) - ay(-1) = b \\ y(1) &= bx(1) - ay(0) = -ay(0) = -ab \\ y(2) &= bx(2) - ay(1) = -ay(1) = a^2 b \\ &\vdots \end{aligned}$$

Secara umum:

$$y(n) = (-1)^n a^n b, \quad n = 0, 1, 2, \ldots, \infty.$$

dengan a dan b tidak nol

Transformasi Z

- Transformasi z untuk sinyal digital $x(n), -\infty < n < \infty$
  dinyatakan dengan deret:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}.$$

- z adalah variabel kompleks yang dalam koordinat polar dapat dinyatakan dengan

$$z = re^{j\theta}$$

- Transformasi Balik-z:

$$x(n) = \mathrm{ZT}^{-1}[X(z)] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz,$$

Sifat-sifat transformasi z

- Linieritas (superposisi):

$$\mathrm{ZT}\,[a_1x_1(n)+a_2x_2(n)] = a_1\,\mathrm{ZT}\,[x_1(n)] + a_2\,\mathrm{ZT}\,[x_2(n)]$$
$$= a_1X_1(z) + a_2X_2(z),$$

where a_1 and a_2 are constants.

- Time-Shifting : y(n) = x(n-k)

$$Y(z) = \mathrm{ZT}\,[x(n-k)] = z^{-k}X(z).$$

- Konvolusi :

$$X(z) = X_1(z)X_2(z).$$

- Derivasi :

$$Z[f(n-k)] = \sum_{n=-\infty}^{\infty} f(u)z^{-(u+k)} = \sum_{u=-\infty}^{\infty} f(u)z^{-u}z^{-k} = z^{-k}\sum_{u=-\infty}^{\infty} f(u)z^{-u} = z^{-k}F(z)$$

Contoh Soal derivasi:

Tuliskan transformasi z dari persamaan berikut:

$$y(n) - 2y(n-1) = 0.5x(n-1)$$

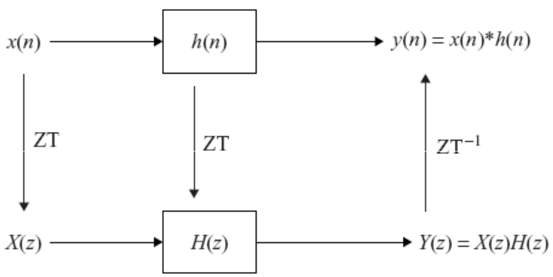
Jawab:

$$Y(z) = 2z^{-1}Y(z) + 0.5z^{-1}X(z)$$

Bisakah menuliskan fungsi transfernya?

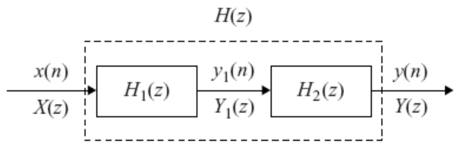
Fungsi Transfer pada Transformasi z

$$H(z) = \frac{Y(z)}{X(z)}.$$



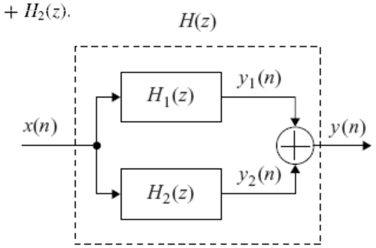
Koneksi Cascade dan Paralel

$$H(z) = H_1(z)H_2(z) = H_2(z)H_1(z).$$



(a)

$$H(z) = H_1(z) + H_2(z).$$



(b)

Table 4.1
Table of z-transforms of signals

Analog Signal	Sampled Signal	Z-transformed Signal
	$A\delta(n)$	A
$Au(t)$	$Au(n)$	$\frac{Az}{z-1}$
$Ae^{-at}u(t)$	$Ae^{-aTn}u(n)$	$\frac{Az}{z-e^{-aT}}$
	$Ac^{n}u(n)$	$\frac{Az}{z-c}$, $c = e^{-aT}$
$Atu(t)$	$AnTu(n)$	$\frac{ATz}{(z-1)^2}$
$A\cos(wt)u(t)$	$A\cos(wTn)u(n)$	$\frac{Az[z-\cos(wT)]}{z^2-2z\cos(wT)+1}$
$A\sin(wt)u(t)$	$A\sin(wTn)u(n)$	$\frac{Az\sin(wT)}{z^2-2z\sin(wT)+1}$
$Ae^{-at}\cos(wt+\alpha)u(t)$	$Ac^{n}\cos(wTn+\alpha)$	$\frac{Az[z\cos(\alpha)-c\cos(\alpha-wT)]}{z^2-2cz\cos(wT)+c^2}$

Contoh menggunakan Tabel Transformasi z

Input pada ADC dengan Perioda Sampling: T=0,5 detik adalah

$$x(t) = 7e^{-3t}u(t)$$

Tuliskan bentuk transformasi dari X(z) :
Jawab: untuk T = 0,5 maka A = 7 dan a = 1,5 sehingga

$$x(n) = 7e^{-1.5n}u(n)$$

$$X(z) = \frac{7z}{z-e^{-1.5}}$$

Example 3.10: The LTI system with transfer function

$$H(z) = 1 - 2z^{-1} + z^{-3}$$

can be factored as

$$H(z) = \left(1 - z^{-1}\right)\left(1 - z^{-1} - z^{-2}\right) = H_1(z)H_2(z).$$

Thus, the overall system $H(z)$ can be realized as the cascade of the first-order system $H_1(z) = 1 - z^{-1}$ and the second-order system $H_2(z) = 1 - z^{-1} - z^{-2}$.

FIR dalam bentuk transformasi z

- Persamaan FIR :

$$y(n) = b_0x(n) + b_1x(n-1) + \cdots + b_{L-1}x(n-L+1)$$
$$= \sum_{l=0}^{L-1} b_l x(n-l).$$

- Dapat bentuk transformasi z

$$Y(z) = b_0X(z) + b_1z^{-1}X(z) + \cdots + b_{L-1}z^{-(L-1)}X(z)$$
$$= \left(b_0 + b_1z^{-1} + \cdots + b_{L-1}z^{-(L-1)}\right)X(z).$$

- Sehingga fungsi transfer untuk FIR:

$$H(z) = b_0 + b_1z^{-1} + \cdots + b_{L-1}z^{-(L-1)} = \sum_{l=0}^{L-1} b_l z^{-l}.$$

IIR dalam bentuk transformasi z

- Persamaan IIR

$$y(n) = b_0x(n) + b_1x(n-1) + \cdots + b_{L-1}x(n-L+1) - a_1y(n-1) - \cdots - a_My(n-M)$$
$$= \sum_{l=0}^{L-1} b_l x(n-l) - \sum_{m=1}^{M} a_m y(n-m).$$

- Dalam bentuk transformasi z :

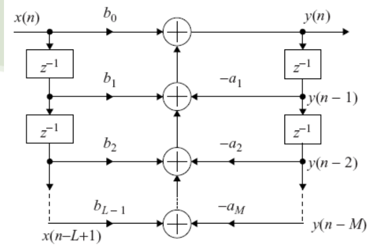
$$Y(z) = b_0X(z) + b_1z^{-1}X(z) + \cdots + b_{L-1}z^{-L+1}X(z) - a_1z^{-1}Y(z) - \cdots - a_Mz^{-M}Y(z)$$
$$= \left(\sum_{l=0}^{L-1} b_l z^{-l}\right) X(z) - \left(\sum_{m=1}^{M} a_m z^{-m}\right) Y(z).$$

- Sehingga fungsi transfer untuk Filter IIR

$$H(z) = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{1 + \sum_{m=1}^{M} a_m z^{-m}} = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{\sum_{m=0}^{M} a_m z^{-m}},$$

Contoh : Perhatikan Moving average filter

$$Y(z) = \frac{1}{L}\sum_{l=0}^{L-1} z^{-l}X(z).$$



Using the geometric series defined in Appendix A, the transfer function of the filter can be expressed as

$$H(z) = \frac{1}{L}\sum_{l=0}^{L-1} z^{-l} = \frac{1}{L}\left(\frac{1-z^{-L}}{1-z^{-1}}\right) = \frac{Y(z)}{X(z)}. \quad (3.43)$$

This equation can be rearranged as

$$Y(z) = z^{-1}Y(z) + \frac{1}{L}\left[X(z) - z^{-L}X(z)\right].$$

Taking the inverse z-transform of both sides, we obtain

$$y(n) = y(n-1) + \frac{1}{L}\left[x(n) - x(n-L)\right].$$

Pole dan Zeros

- Persamaan Filter IIR dapat difaktorisasi menjadi

$$H(z) = b_0\frac{\prod_{l=1}^{L-1}(z - z_l)}{\prod_{m=1}^{M}(z - p_m)} = \frac{b_0(z-z_1)(z-z_2)\cdots(z-z_{L-1})}{(z-p_1)(z-p_2)\cdots(z-p_M)}.$$