

# Process Dynamic Modelling

Dr. Ir. Yeffry Handoko Putra, M.T

## Session Outlines & Objectives

### Outlines

- ❑ What is dynamic modeling?
- ❑ Why do we need dynamic models?
- ❑ Dynamic modeling methods
- ❑ Mathematical dynamic modeling of controlled processes

### Objectives

- ❑ The importance of process dynamic
- ❑ Know the methods used to model a process
- ❑ Know various type of common mathematical dynamic model of industrial process

## Dynamics vs. Steady State Model

- ❑ Chemical process are generally designed from a steady-state point-of-view
  - Steady state model
    - Steady state: No further changes in all variables
    - No dependency in time: No transient behavior
- ❑ Chemical processes are dynamically changing continuously
  - Dynamics is the time varying behavior of processes
- ❑ Steady-state change indicates where the process is going and the dynamic characteristics of a system indicates what dynamic path it will take

## Why do we need dynamic model?

- ❑ People's emotion: An Everyday Example of Dynamic Modelling

**Do we give the same approach to handle both situations?**

- What is your objective?



## Why do we need dynamic model?

### Do the bus and bicycle have different dynamics?

- Which can make a U-turn in 1.5 meter?
- Which responds better when it hits a bump?



Dynamic performance depends more on the vehicle than the driver!

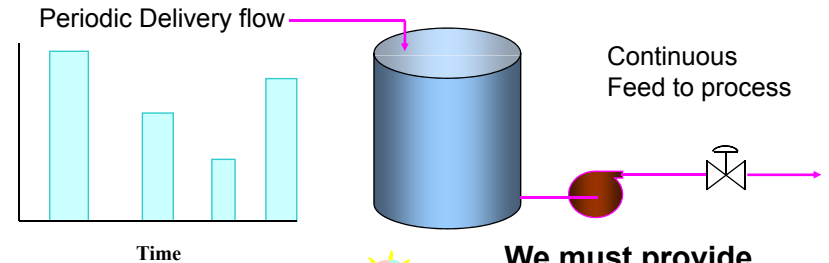


The process dynamics are important than the computer control!

## Why do we need dynamic model?

### Feed material is delivered periodically, but the process requires a continuous feed flow

- How large should the tank volume be?

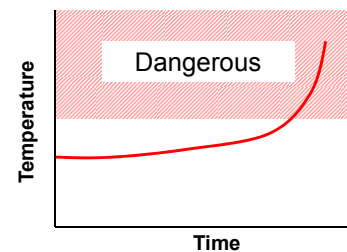
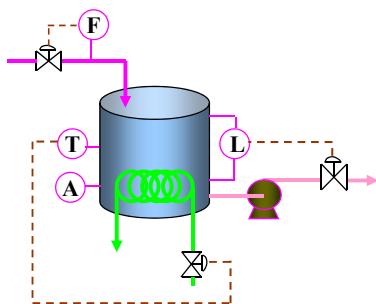


We must provide process flexibility for good dynamic performance!

## Why do we need dynamic model?

### The cooling water pumps have failed

- How long do we have until the exothermic reactor runs away?



Process dynamics are important for safety!

## Why do we need dynamic model?

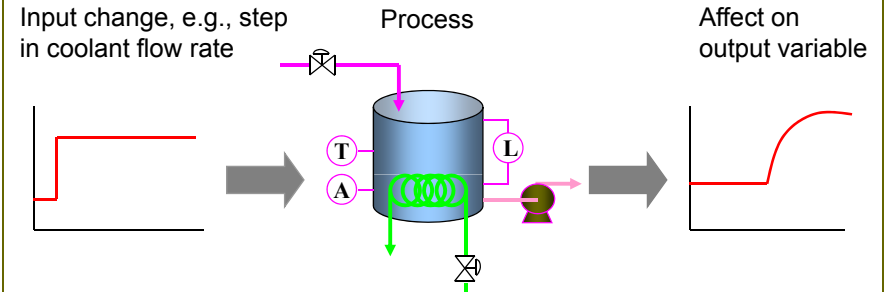
- In order to be able to design a controller, to achieve a specific objective, for the system under concern we **must** know which system we are dealing with

- The process dynamics are important than the computer control
  - To improve understanding of the process
  - To select the right controller
- We must provide process flexibility for good dynamic performance
  - To optimize process design/operating conditions
  - To accommodate the possibility of the change of process dynamic  
→ **ROBUST!**
- Process dynamics are important for safety

## How to Obtain a Dynamic Model?

- ❑ Model can be described using verbal, table, mathematic, etc.
- ❑ The rationale for mathematical modeling
  - To improve understanding of the process
  - To train plant operating personnel
  - To design the control strategy for a new process
  - To design the control law
  - To select the controller setting
  - To optimize process operating conditions
- ❑ Main mathematically modeling methods:
  - Physical modeling
    - Based on physicochemical law
  - Identification
    - Based on process data analysis
  - Semi-empirical models
    - Combined approach

## Why do we develop mathematical models?



How does the process input influence the response?

- How far?
- How fast
- "Shape"

Math models help us answer these questions!

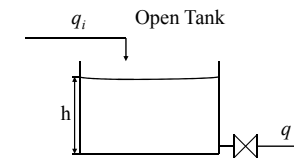
## Physical modelling (1)

- ❑ Total mass balance
 
$$[\text{Rate of mass accumulation within CV}] = [\text{Rate of mass in from surroundings}] - [\text{Rate of mass out to surroundings}]$$
- ❑ Component mass (molar) balance
 
$$[\text{Rate of mass accumulation within CV}] = [\text{Rate of mass in from surroundings}] - [\text{Rate of mass out to surroundings}] + [\text{Rate of mass creation within CV}]$$
- ❑ Total energy (enthalpy) balance
 
$$[\text{Rate of energy accumulation within CV}] = [\text{Rate of energy in from surroundings}] - [\text{Rate of energy out to surroundings}]$$



## Physical modelling (2)

- ❑ Illustrative Example: Open Liquid Storage Tank



$$\begin{aligned} \text{Total mass inside CV} &= \rho Ah \\ \text{Rate of Mass Accumulation inside CV} &= \frac{d(\rho Ah)}{dt} \\ \text{Rate of mass into CV by flow} &= \rho q_i \\ \text{Rate of mass out of CV by flow} &= \rho q \end{aligned}$$

$$\frac{d(\rho Ah)}{dt} = \rho q_i - \rho q \Rightarrow \rho A \frac{dh}{dt} = \rho q_i - \rho q$$



## Physical modelling (3)

- ❑ Follows conservation laws
- ❑ Requires a specific expertise
- ❑ Difficult to develop
- ❑ Expensive
- ❑ Time consuming
- ❑ Too complex model
- ❑ Found in the fields that need accurate model, e.g. aerospace shuttle, aircraft



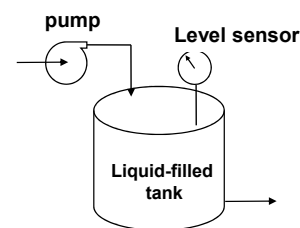
## Identification

- ❑ Based on the operation data
- ❑ Easy to develop
- ❑ Requires well designed experimental data
- ❑ The behavior is correct only around the experimental condition
- ❑ Obtained model obtained usually quite simple for control purpose
- ❑ Found in the industrial process control area

## Process Dynamic Characteristics

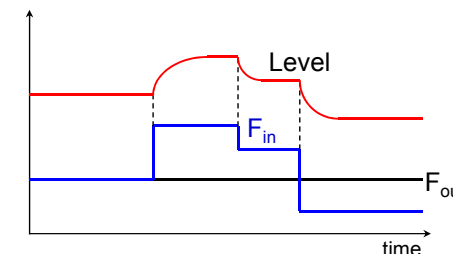
- ❑ Self-regulating and non self-regulating
- ❑ Lag
  - 1<sup>st</sup> order
  - 2<sup>nd</sup> order
  - Higher order
- ❑ Dead time
- ❑ Interacting and non interacting

## Self-regulating process

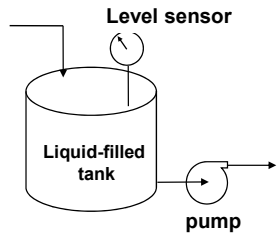


$$\rho \frac{dV}{dt} = \rho A \frac{dL}{dt} = \rho F_{in} - \rho F_{out}$$

- In flow is set independent of level
- Out flow is dependent of level

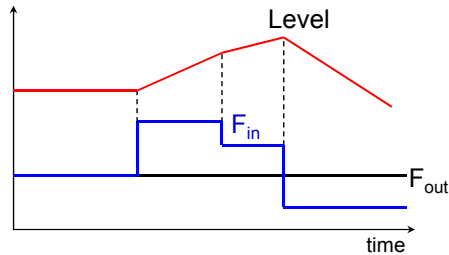


## Non Self-regulating process



$$\rho \frac{dV}{dt} = \rho A \frac{dL}{dt} = \rho F_{in} - \rho F_{out}$$

- In flow and out flow are set independent of level
- Also known as Integrating Processes



## 1<sup>st</sup> Order Processes (1)

The basic equation is:

$$\tau_p \frac{dy(t)}{dt} + y(t) = K_p u(t) \quad \square \text{ Differential equation}$$

$$G_p(s) = \frac{K_p}{\tau_p s + 1} \quad \square \text{ Transfer function}$$

$K_p$  = steady-state gain

$\tau_p$  = time constant

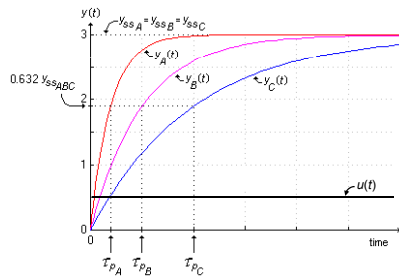
- Note that gain and time constant define the behavior of a 1<sup>st</sup> order process.

## 1<sup>st</sup> Order Processes (2)

- Same  $K_p$ , different  $\tau_p$

$$K_{pA} = K_{pB} = K_{pC}$$

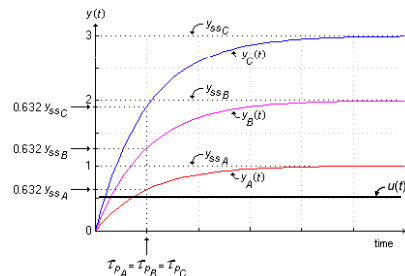
$$\tau_{pA} < \tau_{pB} < \tau_{pC}$$



- Different  $K_p$ , same  $\tau_p$

$$K_{pA} < K_{pB} < K_{pC}$$

$$\tau_{pA} = \tau_{pB} = \tau_{pC}$$



Always overdamped

## 1<sup>st</sup> Order Processes (3)

Balance	Input	Output	$K_p$	$\tau$
Component material	$C_{A0}$	$C_A$	$\frac{F}{F + Vk}$	$\frac{V}{F + Vk}$
Energy	$T_0$	$T$	1.0	$\frac{V}{F}$
Overall material	$F$	$L$	$\frac{1}{0.5kL_p - 0.5}$	$\frac{A}{0.5kL_p - 0.5}$
Current	$E_0$	$E$	1.0	$RC$
Force	$z_0$	$z$	1.0	$f/k$

## 2<sup>nd</sup> Order Processes (1)

The basic equation is:

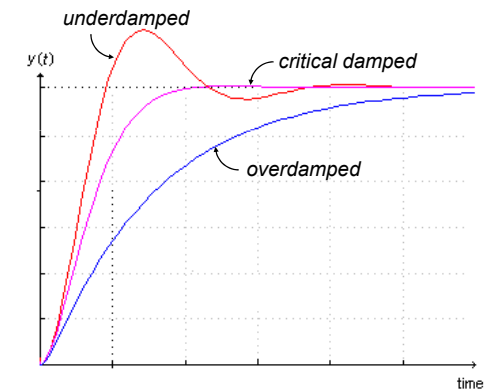
$$\tau_p^2 \frac{d^2 y(t)}{dt^2} + 2\zeta \tau_p \frac{dy(t)}{dt} + y(t) = K_p u(t) \quad \square \text{ Differential equation}$$

$$G_p(s) = \frac{K_p}{\tau_p^2 s^2 + 2\zeta \tau_p s + 1} \quad \square \text{ Transfer function}$$

- Note that the gain, time constant, and the damping factor define the dynamic behavior of 2<sup>nd</sup> order process

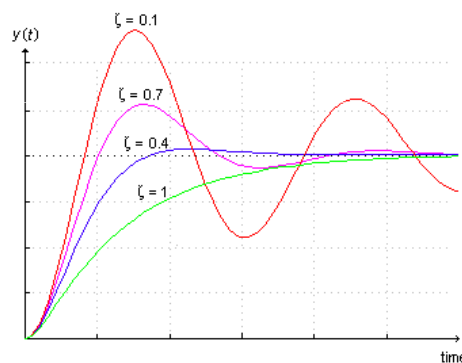
## 2<sup>nd</sup> Order Processes (2)

- Underdamped vs. Overdamped



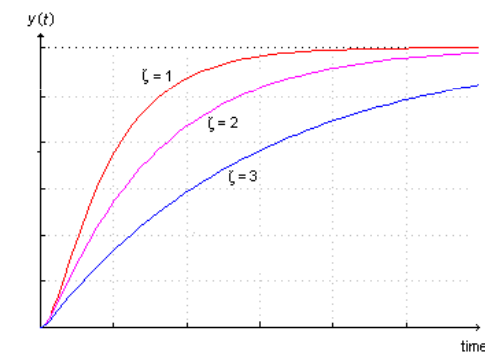
## 2<sup>nd</sup> Order Processes (3)

- Effect of  $\zeta$  on Underdamped Response



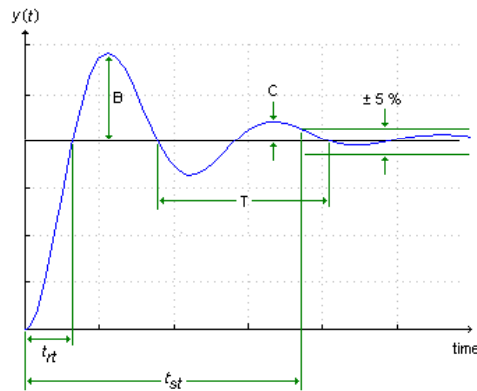
## 2<sup>nd</sup> Order Processes (4)

- Effect of  $\zeta$  on Overdamped Response



## 2<sup>nd</sup> Order Processes (5)

### Characteristics of an Underdamped Response



- Rise time
- Overshoot (B)
- Decay ratio (C/B)
- Settling or response time
- Period (T)

## 2<sup>nd</sup> Order Processes (6)

	Balance	Input	Output	$K_p$	$\tau^2$	$2\xi\tau$
Component material	$C_{A0}$	$C_B$		$\frac{VK}{F+VK}$	$\tau_A\tau_B$	$\tau_A + \tau_B$
	Energy	$T_0$	$T$		[see question 5.2]	
Overall material	$F$	$L$		$\frac{1}{0.5kL_s - 0.5}$	$\left[\frac{A}{0.5kL_s - 0.5}\right]^2$	$2\tau$
	Current	$E_0$	$E$	1.0	LC	RC
	Force	$h$	$z$	$1/k'$	$m/k'$	$f/k'$

## Underdamped Processes

- Many examples can be found in mechanical and electrical system
- Among chemical processes, open-loop underdamped process is quite rare
- However, when the processes are controlled, the responses are usually underdamped
- Depending on the controller tuning, the shape of response will be decided
- Slight overshoot results short rise time and often more desirable
- Excessive overshoot may results long-lasting oscillation

## Dead Time

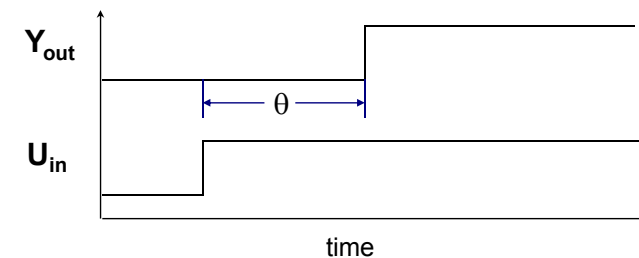
### Fluid transportation through a pipe

- Also called as distance-velocity lag, transportation lag, time delay

$\theta = \text{dead time}$

$$Y_{out}(t) = U_{in}(t - \theta)$$

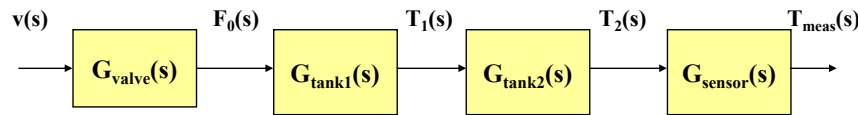
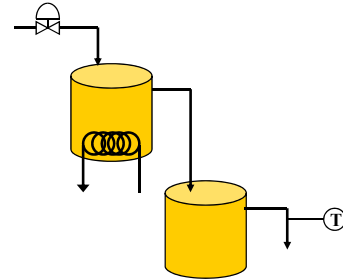
$$Y_{out}(s) = e^{-\theta s} U_{in}(s)$$



## Structure of Process Systems (1)

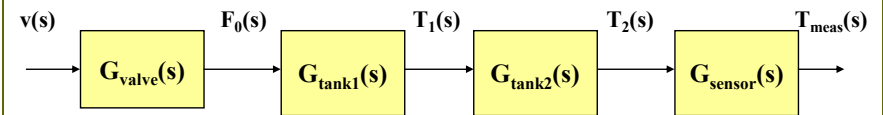
### Non-Interacting Series

- ❑ The output from an element does not influence the input to the same element
- ❑ Common example is tanks in series with pumped flow between
- ❑ Block diagram as shown



## Structure of Process Systems (2)

### Non-Interacting Series



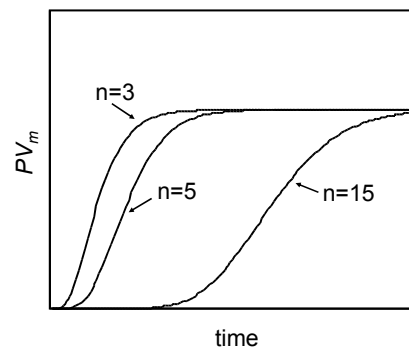
In general: 
$$\frac{Y(s)}{X(s)} = \prod_{i=1}^n G_i(s)$$

With each element a first order system: 
$$\frac{Y(s)}{X(s)} = \prod_{i=1}^n \frac{K_i}{(\tau_i s + 1)}$$

- overall gain is product of gains
- no longer first order system
- slower than any single element

## Structure of Process Systems (3)

### High Order Processes



- ❑ The larger n, the more sluggish the process response (i.e., the larger the effective deadtime)

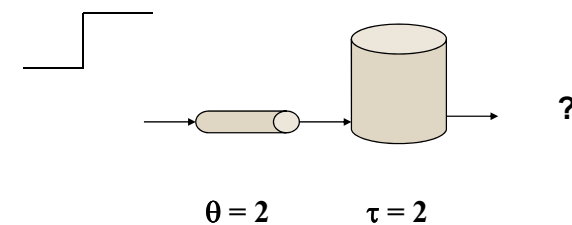
- ❑ Transfer function:

$$G_p(s) = \frac{K_p}{(\tau_p s + 1)^n}$$

## Structure of Process Systems (4)

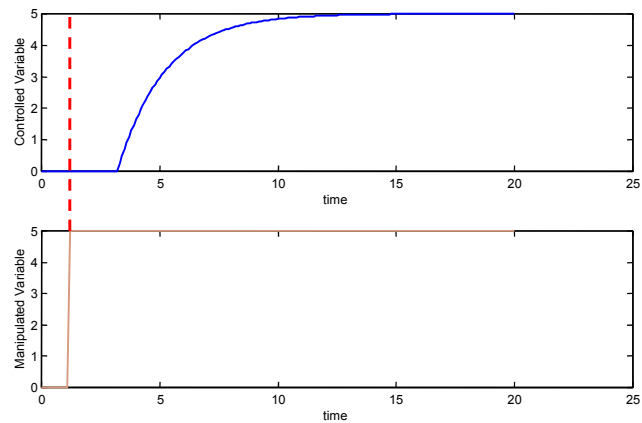
### Exercise:

Sketch the step response for the system below





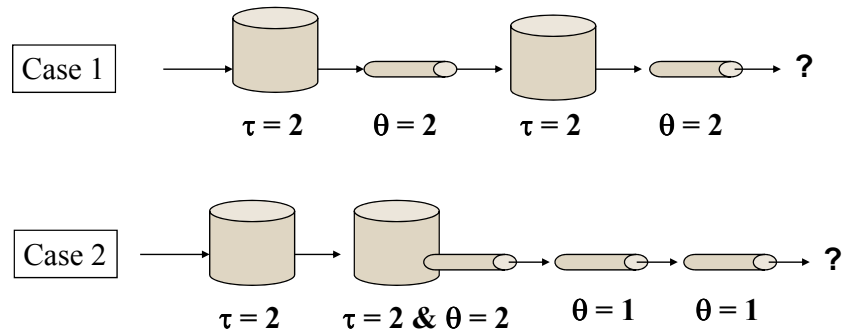
## Structure of Process Systems (5)



## Structure of Process Systems (6)

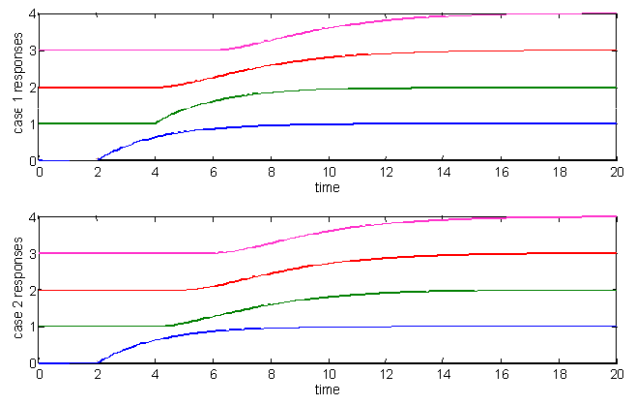
### Exercise:

Sketch the step response for each of the systems below and compare the results



## Structure of Process Systems (7)

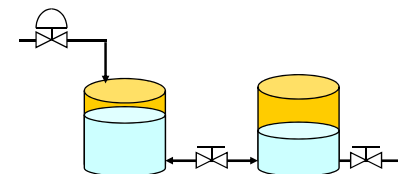
- Two plants can have different intermediate variables and have the same input-output behavior!



## Structure of Process Systems (8)

### Interacting process

- Many chemical processes exhibit interacting nature
- The output from an element influence the input to the another element and vice versa





## Session Summary

- ❑ A good understanding on controlled process dynamic is very important to achieve a good control
- ❑ Process models can be obtained using physical, identification and combined approaches
- ❑ Many industrial processes can be represented by simple mathematical process dynamic, i.e. 1<sup>st</sup> order, 2<sup>nd</sup> order (with dead time)