

# MATAKULIAH SISTEM DIGITAL

## PERTEMUAN V

# RANGKAIAN ARITMATIK

OLEH :  
HIDAYAT

JURUSAN TEKNIK KOMPUTER  
UNIKOM  
2012

# Penjumlahan bil. biner

- Mari kita hitung :

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = \textcolor{red}{1}0; 0 \text{ sbg hasil dan } \textcolor{red}{1} \text{ sbg carry}$$

$$1 + 1 + 1 = \textcolor{red}{1}1; 1 \text{ sbg hasil dan } \textcolor{red}{1} \text{ sbg carry}$$

# Contoh

$$\begin{array}{r} 011 \text{ (3)} \\ + 110 \text{ (6)} \\ \hline 1001 \text{ (9)} \end{array}$$

$$\begin{array}{r} 1001 \text{ (9)} \\ + 1111 \text{ (15)} \\ \hline 11000 \text{ (24)} \end{array}$$

$$\begin{array}{r} 11.011 \text{ (3.375)} \\ + 10.110 \text{ (2.750)} \\ \hline 110.001 \text{ (6.125)} \end{array}$$

Add the following pairs of binary numbers.

- (a) 10110 + 00111   (b) 011.101 + 010.010   (c) 10001111 + 00000001

# Bilangan tak bertanda

- Bilangan yang tidak memiliki tanda ‘+’ ataupun ‘-’
- Contoh :

bilangan 8 bit : 0000000 - 1111111

: 00 - FF<sub>H</sub>

: 0 - 255

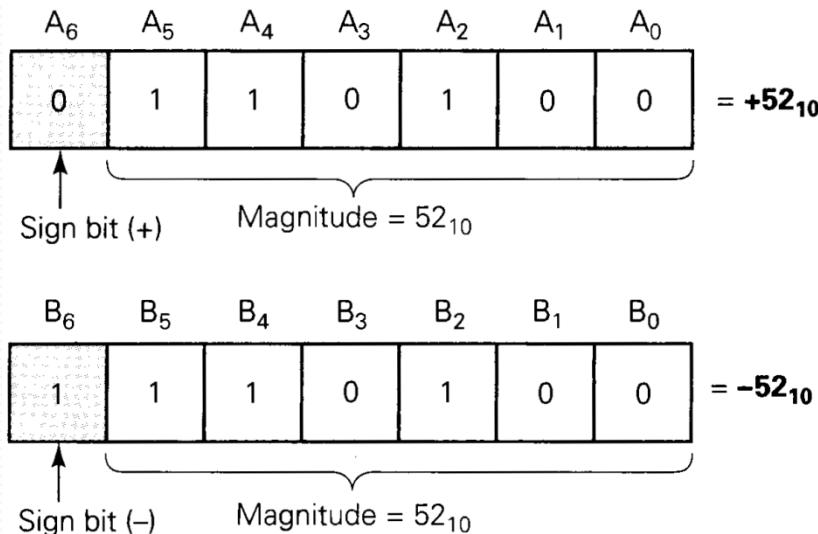
Penjumlahah pada bil. tak bertanda dpt mengakibatkan *overflow* sehingga dibutuhkan *bit carry*.

# Bilangan Bertanda

- Fungsi: untuk merepresentasikan nilai positif (+) dan negatif (-).
- Representasi bilangan bertanda:
  - Sign Magnitude System (bit paling “**besar**” sebagai tanda)
  - Complement 1
  - Complement 2

# Sign Magnitude System

- Bilangan yang memiliki tanda ‘+’ ataupun ‘-’
- Bit MSB : tanda ‘+’ ataupun ‘-’
- ‘0’ : tanda positif, dan
- ‘1’ : tanda negatif.



Contoh :

$$+38 = \textcolor{red}{0} 0100110$$

$$-38 = \textcolor{red}{1} 0100110$$

$$+25 = \textcolor{red}{0} 00000011001$$

$$-25 = \textcolor{red}{1} 00000011001$$

# Bilangan bertanda

- Walaupun mudah dalam memberikan tanda positif dan negatif pada suatu bilangan namun sistem ini untuk diimplementasinya sangat komplek, sehingga sistem ini tidak diimplementasikan

# Bilangan komplement 1

- Komplemen 1 suatu bilangan biner diperoleh dengan cara membalikkan nilai bil. tersebut.

Contoh :  $101101 \rightarrow 010010$  (komplemen 1)

1	0	1	1	0	1
↓	↓	↓	↓	↓	↓
0	1	0	0	1	0

bilangan asli

hasil komplemen 1

# Bilangan komplementen 1

Contoh :

$$\begin{array}{r} (+5) \\ + (+2) \\ \hline (+7) \end{array}$$

$$\begin{array}{r} 0101 \\ + 0010 \\ \hline 0111 \end{array}$$

$$\begin{array}{r} (-5) \\ + (+2) \\ \hline (-3) \end{array}$$

$$\begin{array}{r} 1010 \\ + 0010 \\ \hline 1100 \end{array}$$

$$\begin{array}{r} (+5) \\ + (-2) \\ \hline (+3) \end{array}$$

$$\begin{array}{r} 0101 \\ + 1101 \\ \hline 10010 \\ \text{---} \\ 0011 \end{array}$$

$$\begin{array}{r} (-5) \\ + (-2) \\ \hline (-7) \end{array}$$

$$\begin{array}{r} 1010 \\ + 1101 \\ \hline 10111 \\ \text{---} \\ 1000 \end{array}$$

# Bilangan komplement 2

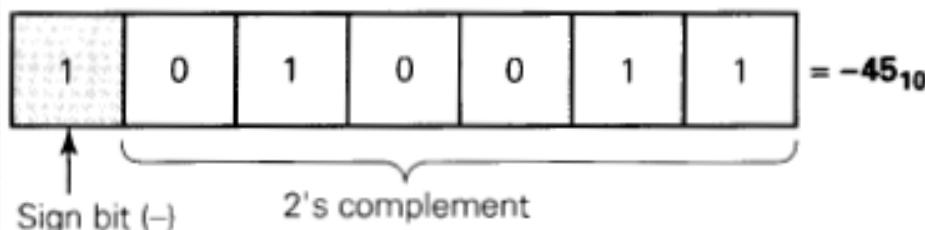
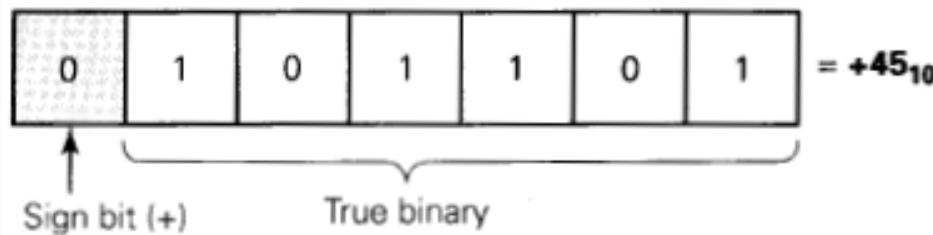
- Komplemen 2 suatu bilangan biner diperoleh dengan cara menambahkan ‘1’ pada LSB hasil komplemen 1 bilangan tersebut.

Contoh :  $101101 \rightarrow 010010 + 1 = 010011$  (kompl. 2)

$$\begin{array}{r} 101101 & \text{bilangan asli} \\ 010010 & \text{komplement 1} \\ + \hline & 1 \\ 010011 & \text{komplement 2} \end{array}$$

# Bilangan komplement 2

- Representasi bilangan bertanda pada komplement 2
  - Jika nilai positif maka nilai bilangan direpresentasikan dalam bentuk aslinya.
  - Jika nilai negatif maka nilai direpresentasikan dalam bentuk komplement 2 .



- Range nilai pada bilangan komplemen 2:  $-2^N$  to  $+(2^N - 1)$

Decimal Value	Signed Binary Using 2's Complement	0	0000
$+7 = 2^3 - 1$	0111	-1	1111
+6	0110	-2	1110
+5	0101	-3	1101
+4	0100	-4	1100
+3	0011	-5	1011
+2	0010	-6	1010
+1	0001	-7	1001
		$-8 = -2^3$	1000

# Penjumlahan pada Bil. Kompl. 2

Contoh :

$$\begin{array}{r} (+5) \\ + (+2) \\ \hline (+7) \end{array}$$

$$\begin{array}{r} 0101 \\ + 0010 \\ \hline 0111 \end{array}$$

$$\begin{array}{r} (-5) \\ + (+2) \\ \hline (-3) \end{array}$$

$$\begin{array}{r} 1011 \\ + 0010 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} (+5) \\ + (-2) \\ \hline (+3) \end{array}$$

$$\begin{array}{r} 0101 \\ + 1110 \\ \hline 10011 \end{array}$$

$$\begin{array}{r} (-5) \\ + (-2) \\ \hline (-7) \end{array}$$

$$\begin{array}{r} 1011 \\ + 1110 \\ \hline 11001 \end{array}$$

ignore

ignore

# Overflow

- Overflow terjadi ketika aritmatika pada komplemen 2 hasilnya salah. (hasil aritmatika melampaui tampungan bit nilai)

Contoh:

$$\begin{array}{r} +9 \rightarrow \\ +8 \rightarrow \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline & 0 \\ \hline & 0 \\ \hline 1 & \underline{0001} \\ \hline \end{array}$$

incorrect sign      ↑      incorrect magnitude

# Perkalian

- a

$$\begin{array}{r} 1001 \\ \times 1011 \\ \hline 1001 \\ 1001 \\ 0000 \\ \hline 1001 \\ \hline 1100011 \end{array} \quad \left. \begin{array}{l} \leftarrow \text{multiplicand} = 9_{10} \\ \leftarrow \text{multiplier} = 11_{10} \\ \\ \text{partial products} \\ \\ \text{final product} = 99_{10} \end{array} \right\}$$

Add  $\left\{ \begin{array}{r} 1001 \\ 1001 \end{array} \right.$   $\left. \begin{array}{l} \leftarrow \text{first partial product} \\ \leftarrow \text{second partial product shifted left} \end{array} \right.$

Add  $\left\{ \begin{array}{r} 11011 \\ 0000 \end{array} \right.$   $\left. \begin{array}{l} \leftarrow \text{sum of first two partial products} \\ \leftarrow \text{third partial product shifted left} \end{array} \right.$

Add  $\left\{ \begin{array}{r} 011011 \\ 1001 \end{array} \right.$   $\left. \begin{array}{l} \leftarrow \text{sum of first three partial products} \\ \leftarrow \text{fourth partial product shifted left} \end{array} \right.$

1100011  $\leftarrow$  sum of four partial products which equals final total product

# Pembagian

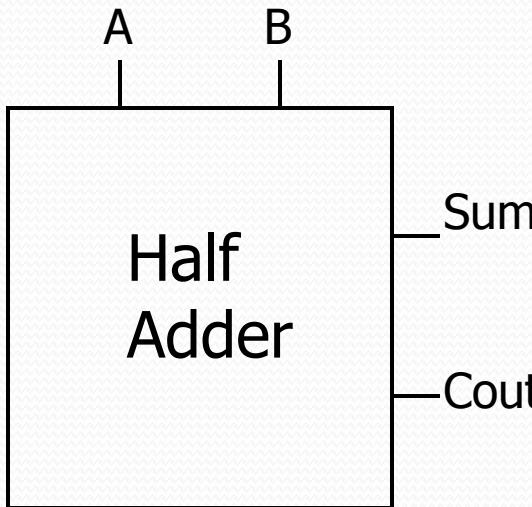
- a

$$\begin{array}{r} 0011 \\ \underline{11} \overline{)1001} \\ 011 \\ \hline 0011 \\ 11 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 0010.1 \\ \underline{100} \overline{)1010.0} \\ 100 \\ \hline 100 \\ 100 \\ \hline 0 \end{array}$$

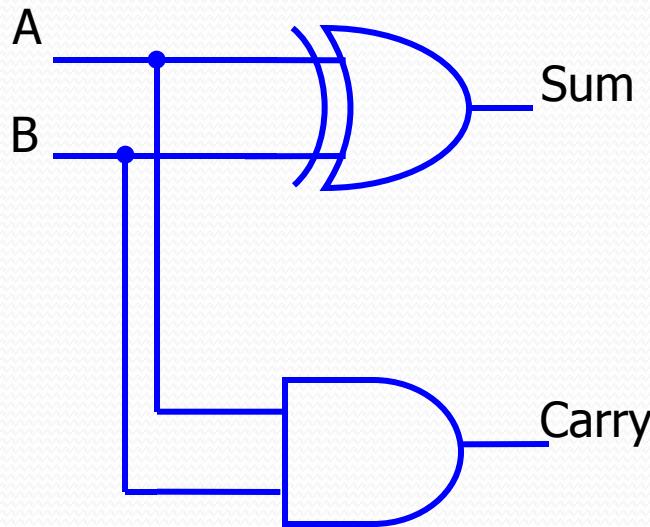
# Rangkaian Aritmatik

# Half Adder (1-bit)



A	B	S(sum)	C(arry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

# Half Adder (1-bit)

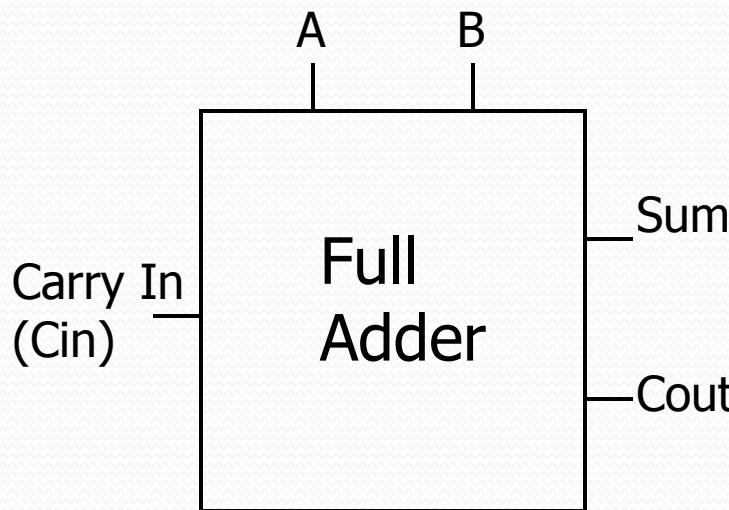


A	B	S(sum)	C(arry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = \overline{A}\overline{B} + A\overline{B} = A \oplus B$$

$$C = AB$$

# Full Adder



Cin	A	B	S(sum)	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

# Full Adder

Cin	AB	00	01	11	10
0	0	0	1	0	1
1	1	1	0	1	0

$$\begin{aligned}
 S &= \text{Cin} \overline{A}\overline{B} + \overline{\text{Cin}}\overline{A}B + \text{Cin}A\overline{B} + \overline{\text{Cin}}A\overline{B} \\
 &= \text{Cin}(\overline{A}\overline{B} + A\overline{B}) + \overline{\text{Cin}}(\overline{A}B + A\overline{B}) \\
 &= \text{Cin}(A \oplus B) + \overline{\text{Cin}}(A \oplus B) \\
 &= \text{Cin} \oplus A \oplus B
 \end{aligned}$$

Cin	AB	00	01	11	10
0	0	0	0	1	0
1	0	0	1	1	1

$$\text{Cout} = \text{Cin}B + \text{Cin}A + AB$$

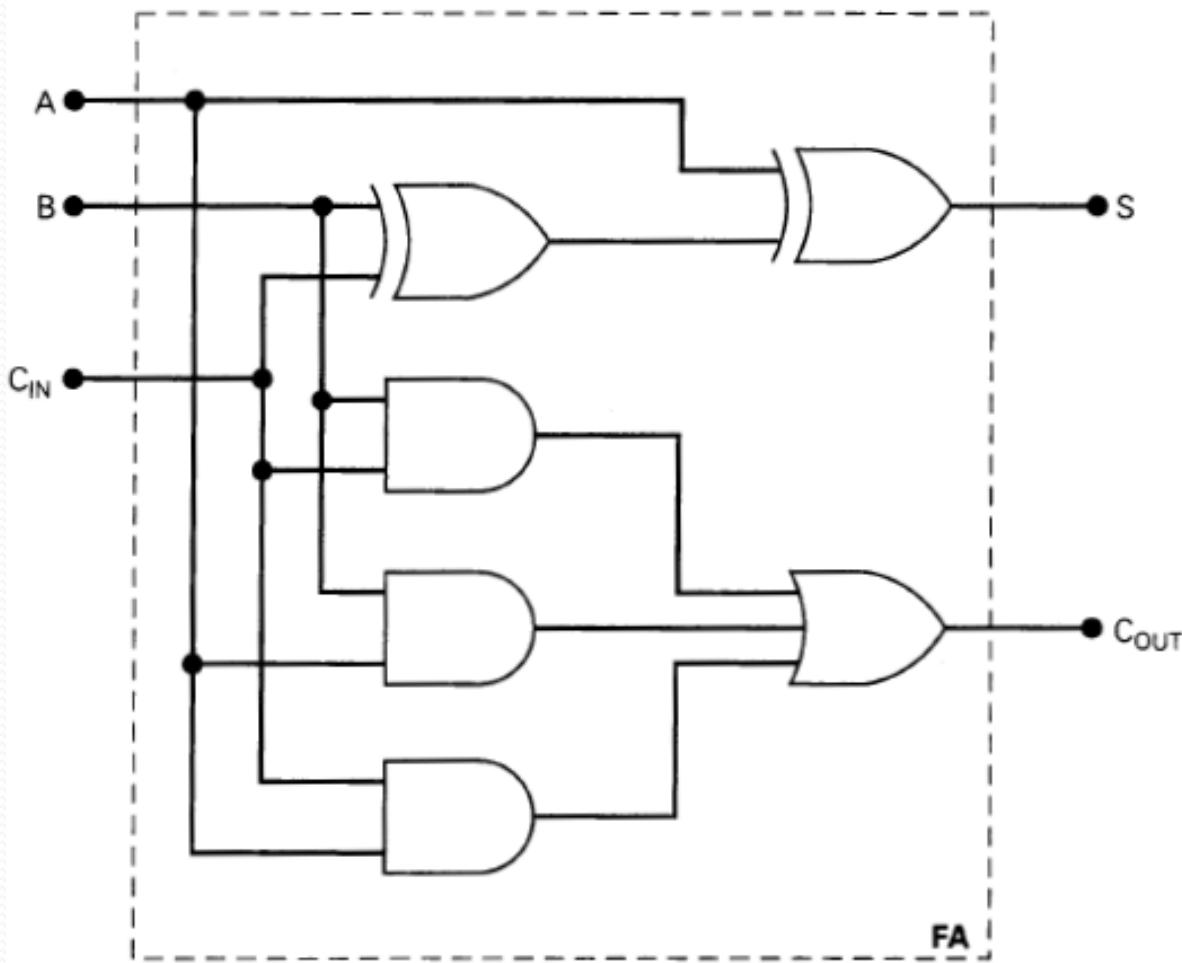
Cin	A	B	S(sum)	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Cin	AB	00	01	11	10
0	0	0	0	1	0
1	0	0	1	1	1

$$\text{Cout} = AB + \text{Cin}(\overline{A}\overline{B} + A\overline{B}) = AB + \text{Cin}(A \oplus B)$$

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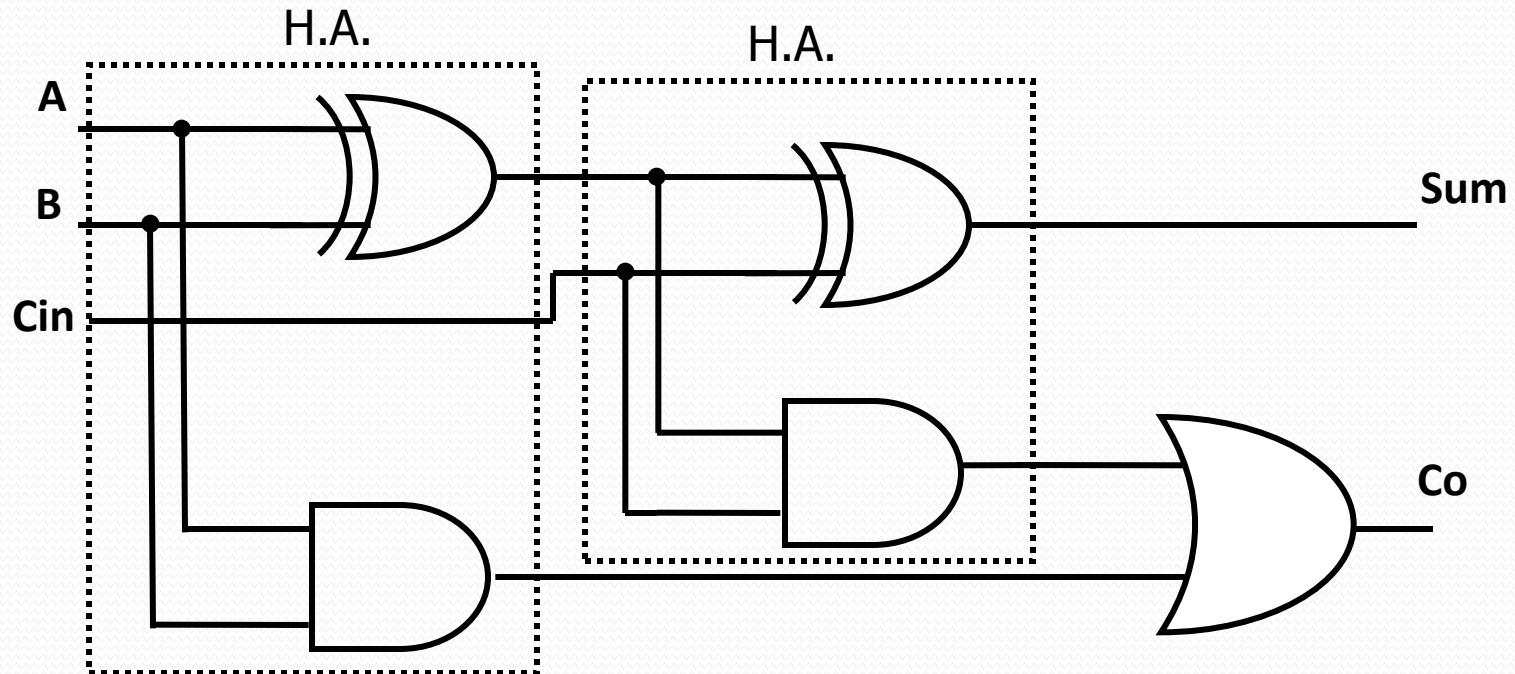
# Full Adder



# Full Adder

$$\text{Sum} = \text{Cin} \oplus A \oplus B$$

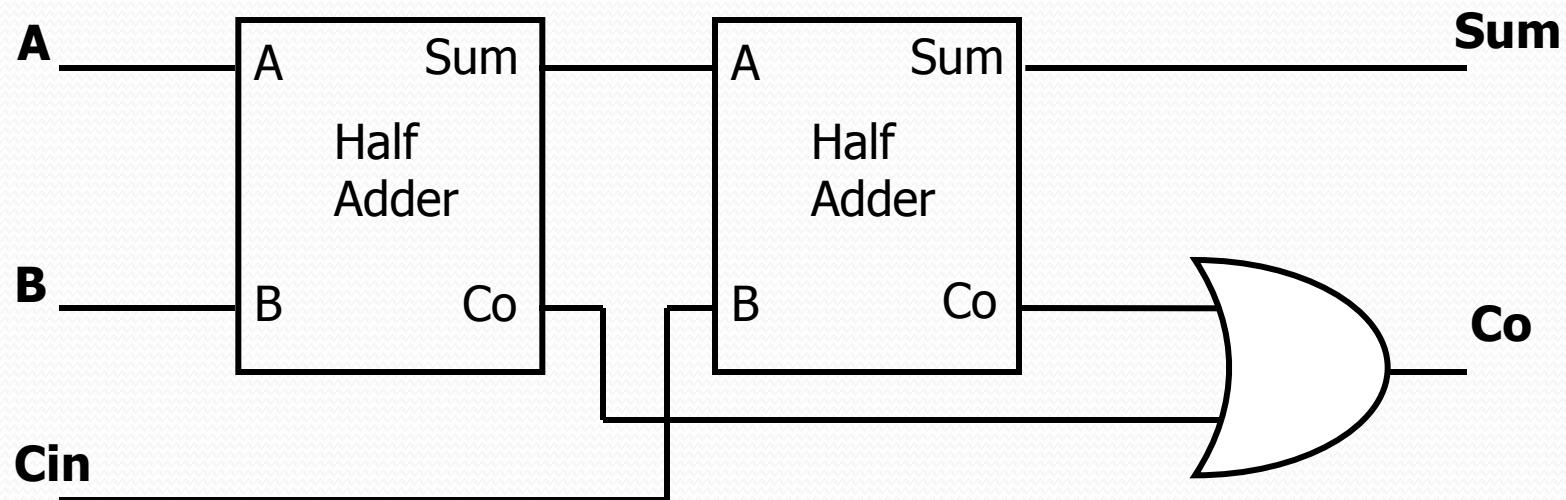
$$\text{Cout} = AB + \text{Cin}(A \oplus B)$$



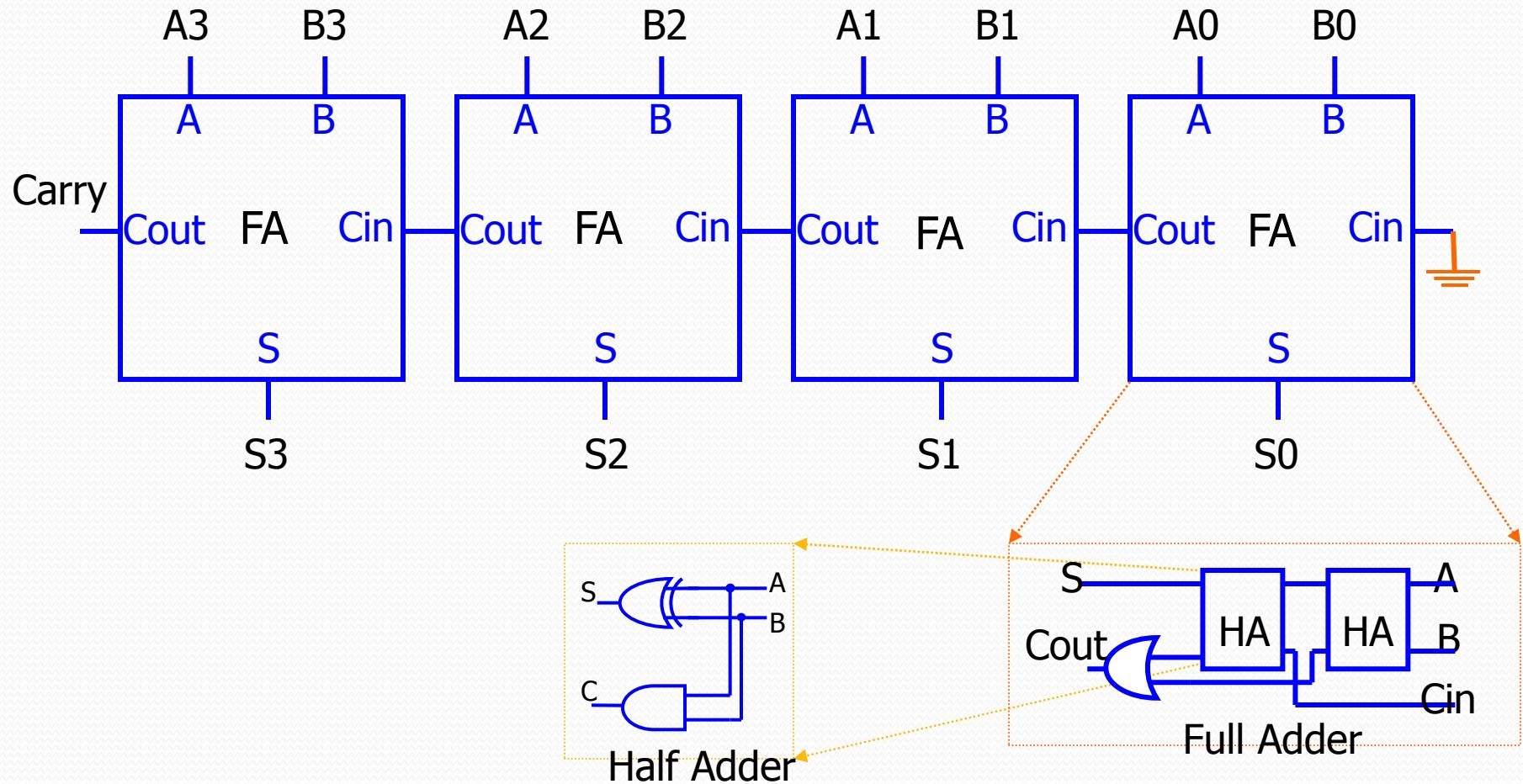
# Full Adder

$$\text{Sum} = \text{Cin} \oplus A \oplus B$$

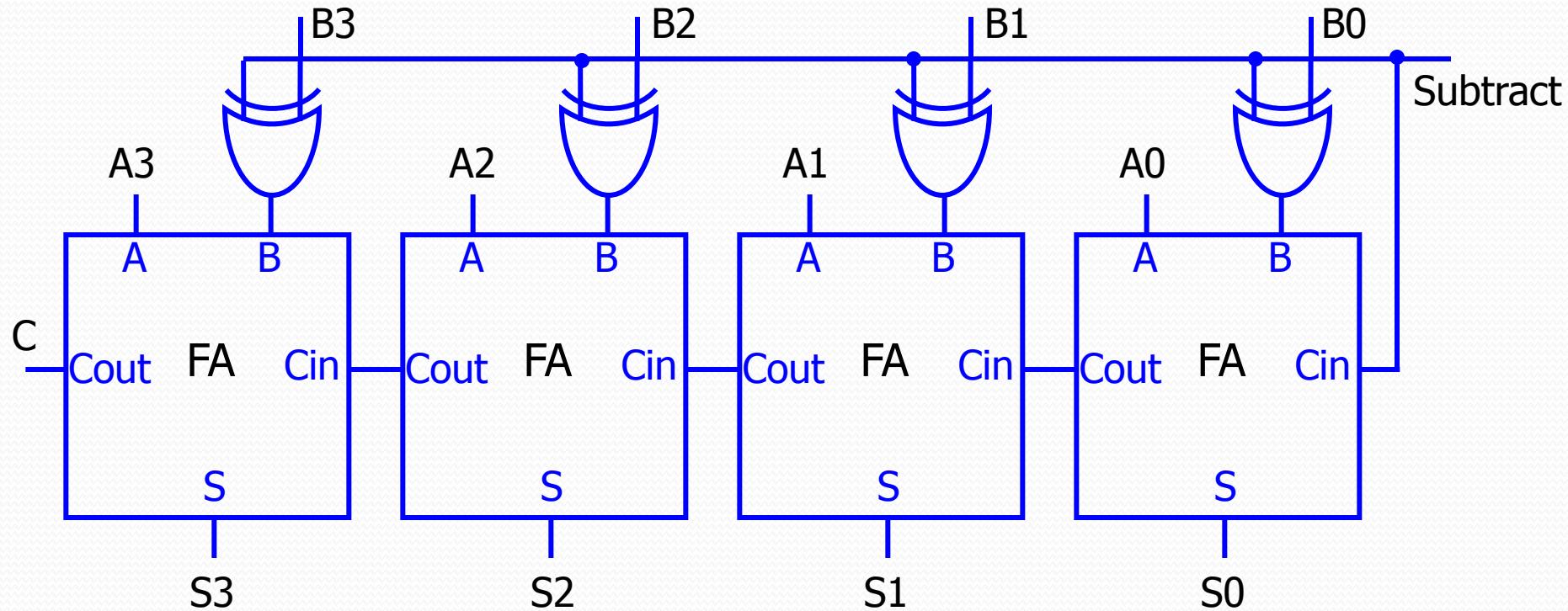
$$\text{Cout} = AB + \text{Cin}(A \oplus B)$$



# 4-bit Ripple Adder menggunakan Full Adder

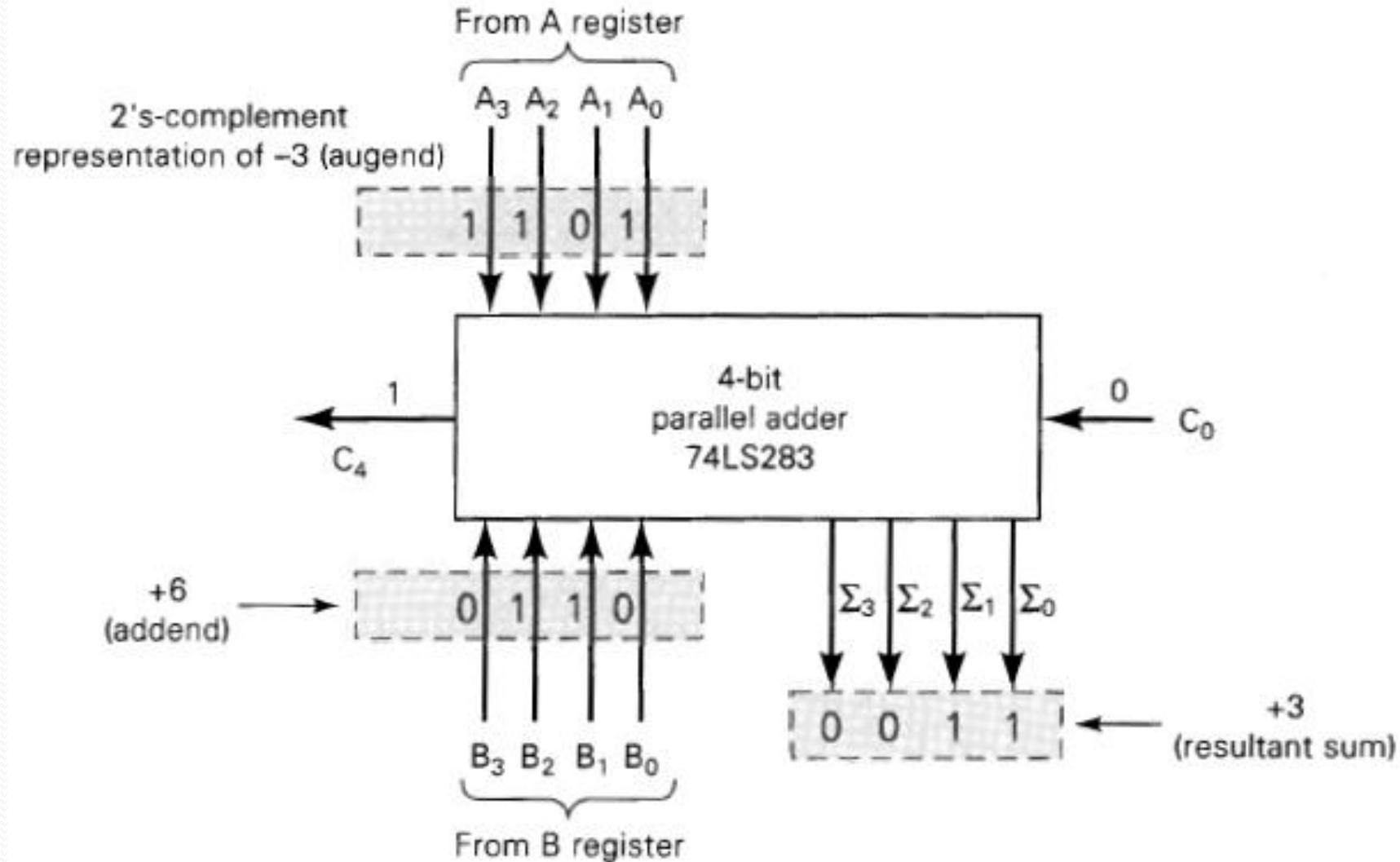


# Disain Subtractor

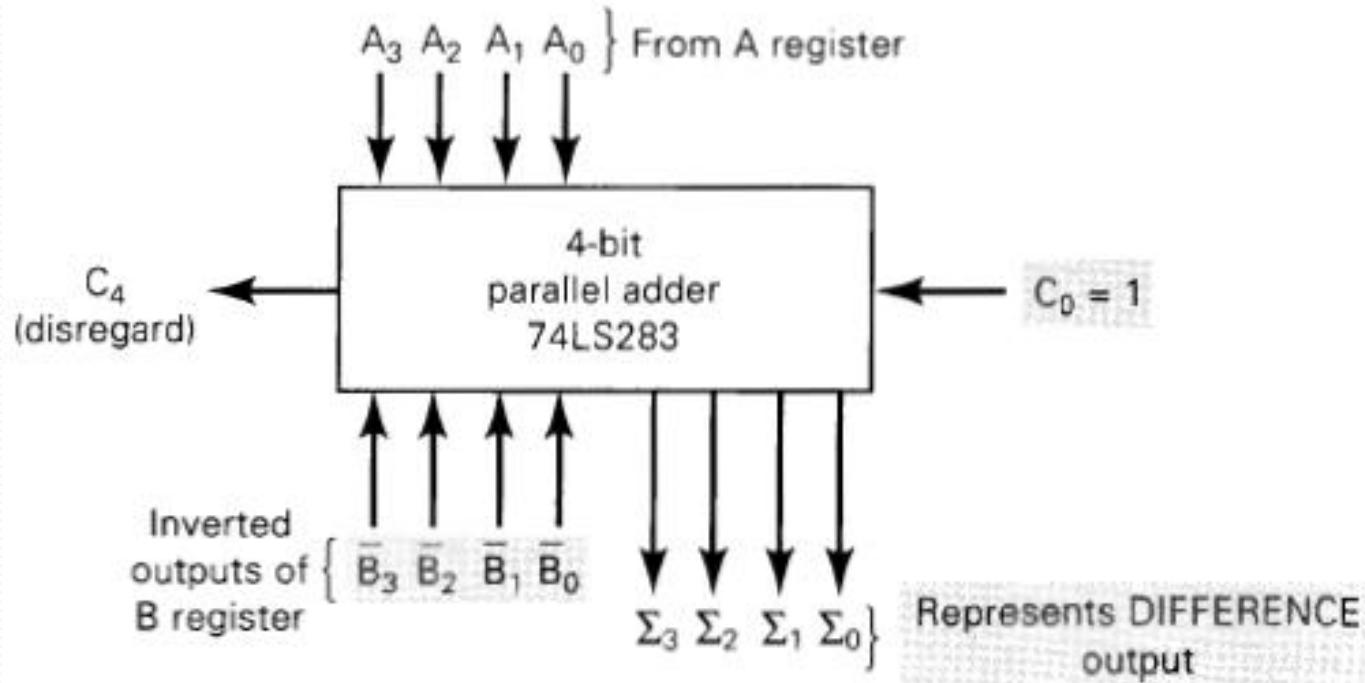


- $A - B = A + (-B)$ 
  - Lakukan komplement 2 pada B
  - Jadikan penjumlahan A dan komplement 2 dari B

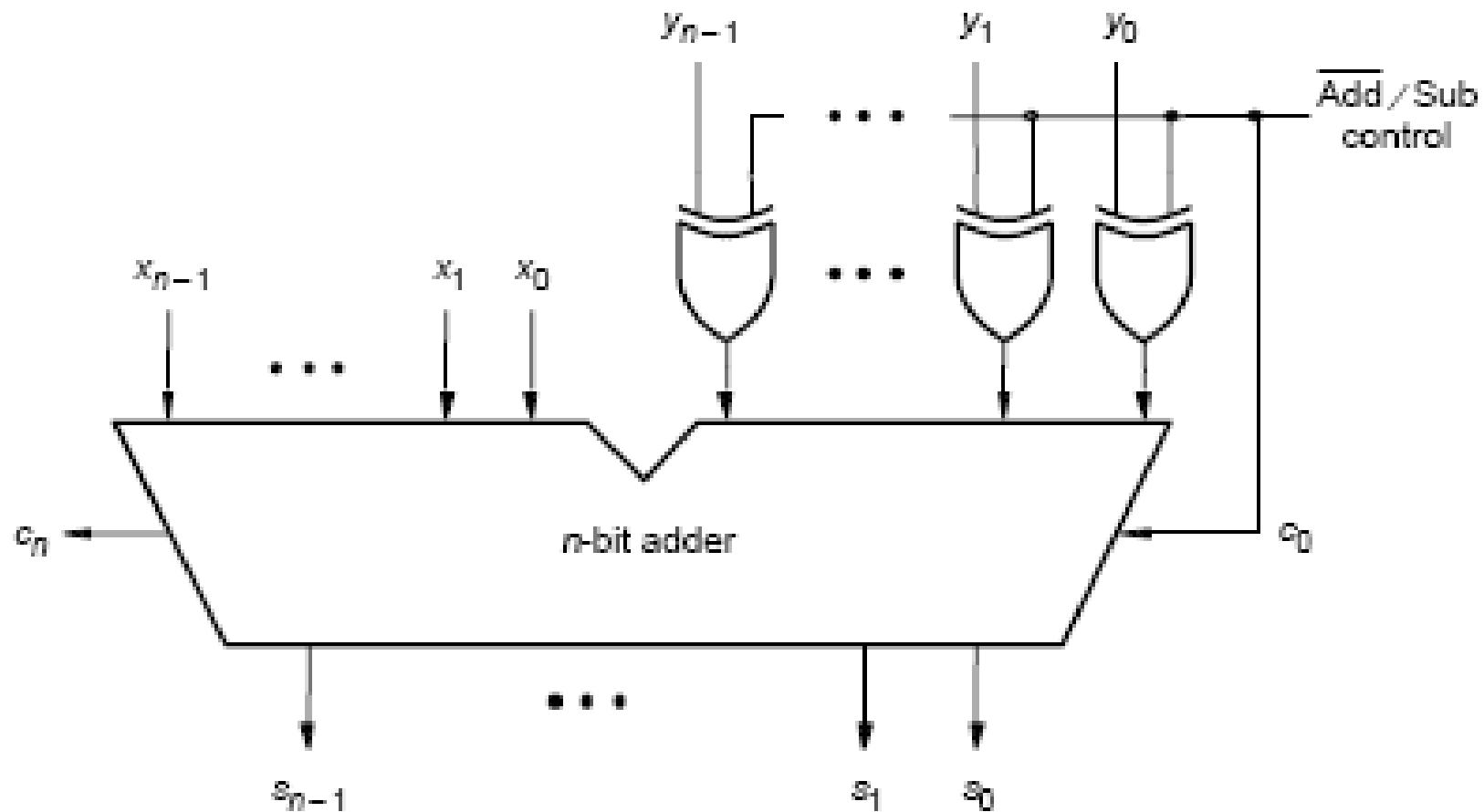
# Parallel adder - Adder



# Parallel adder - Subtractor



# UNIT ADDER/SUBSTRACTOR





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