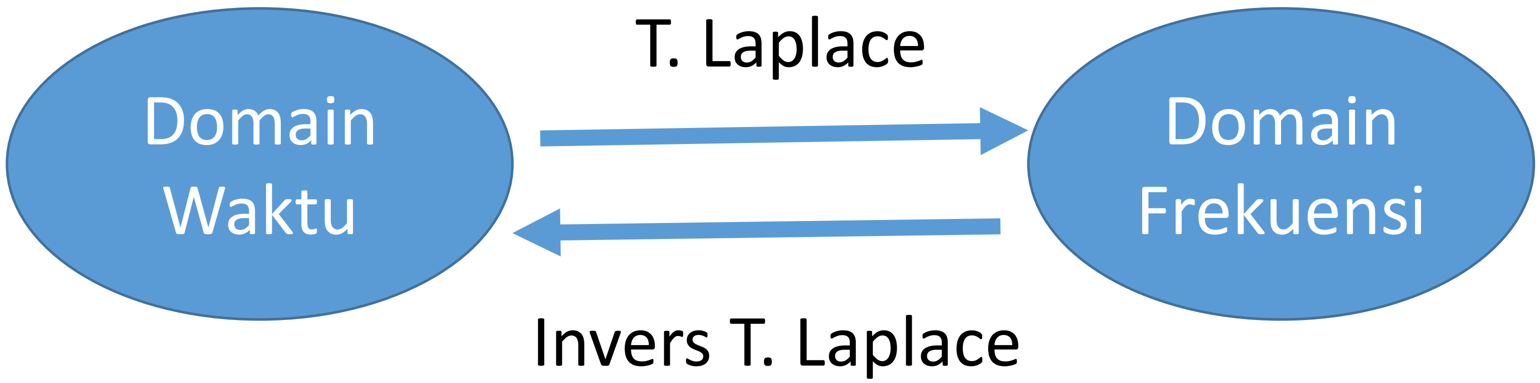
Transformasi Laplace

Signal and System : Schaum Series
Sinyal dan Sistem Jilid 1 dan Jilid 2 : Alan V Oppenheim, Alan S Willsky, S. Hamid Nawab


Domain
Waktu
T. Laplace
Domain
Frekuensi
Invers T. Laplace

Transformasi Laplace

- Transformasi Laplace memberikan karakterisasi yang lebih umum untuk sistem linier waktu kontinyu tidak berubah terhadap waktu (LTI) dan interaksinya dengan sinyal dibandingkan Fourier
- Dapat diterapkan pada analisa sistem tidak stabil, oleh karena itu memegang peranan penting dalam pengujian stabilitas atau ketidakstabilan suatu sistem.
- Transformasi Laplace ada untuk sinyal-sinyal yang tidak mempunyai TF

Transformasi Laplace Bilateral

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

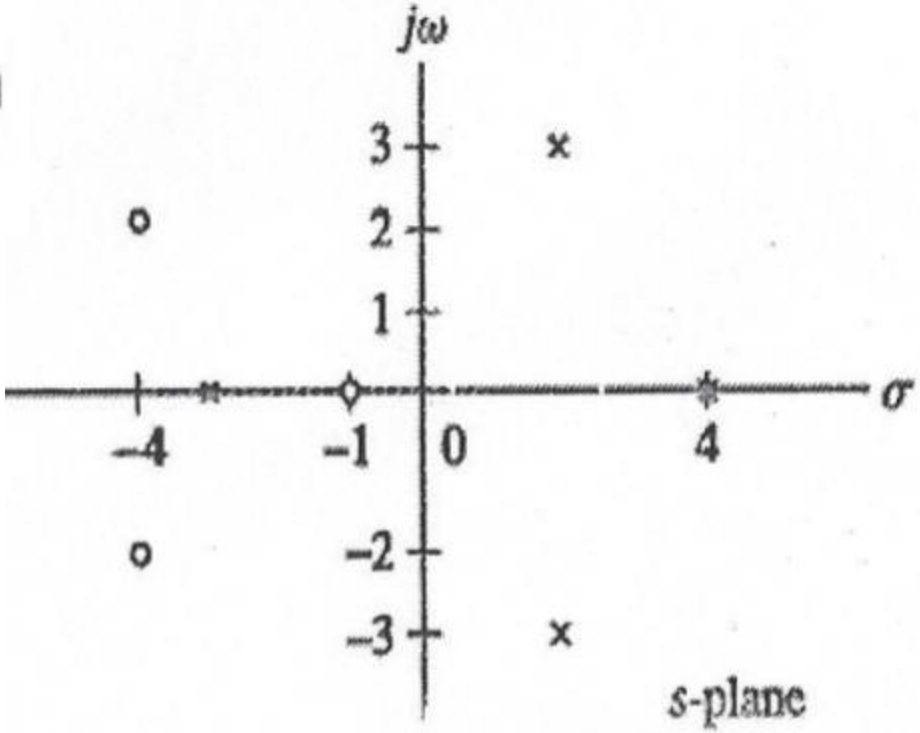
Transformasi Laplace Unilateral (Untuk Sistem Kausal, Untuk menghitung respon sistem kausal yang dinyatakan dengan persamaan differential dengan kondisi awal tidak sama dengan nol

$$X_I(s) = \int_{0^-}^{\infty} x(t) e^{-st} \, dt$$

$$s = \sigma + j\omega$$

Bidang s

- Menggambarkan frekuensi kompleks s secara graphic didalam sebuah bidang kompleks yang disebut bidang s



- Sumbu Horizontal: σ
- Sumbu Vertical: $j\omega$
- $X(j\omega) = X(s)|_{\sigma=0}$

$$X(s) = \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n} = \frac{a_0}{b_0} \frac{(s - z_1) \cdots (s - z_m)}{(s - p_1) \cdots (s - p_n)}$$

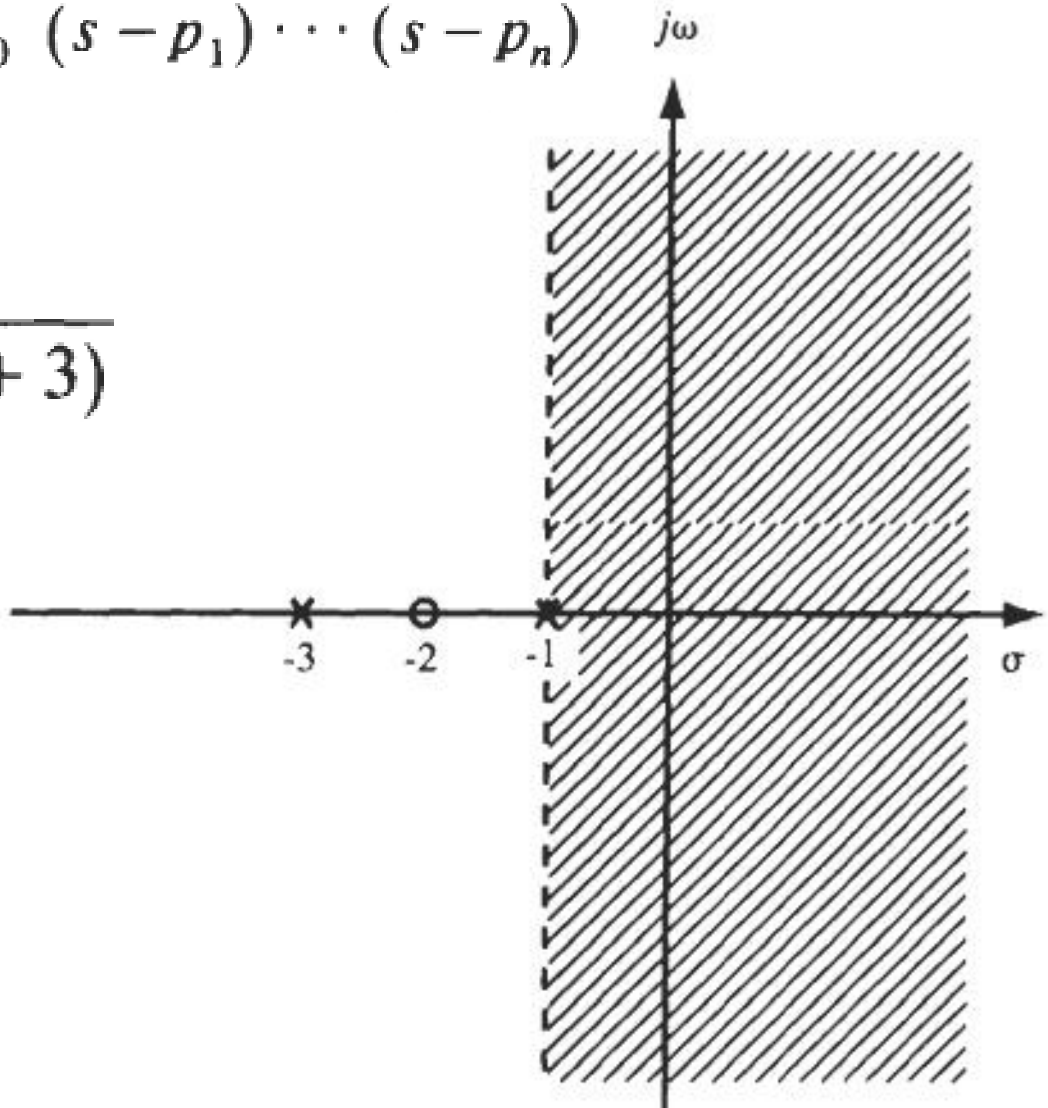
- Zero di $s = -1$ and $s = -4 \pm j2$
- Pole di $s = -3, s = 2 \pm j3, s = 4$

Poles dan Zero

$$X(s) = \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n} = \frac{a_0}{b_0} \frac{(s - z_1) \cdots (s - z_m)}{(s - p_1) \cdots (s - p_n)}$$

$$X(s) = \frac{2s + 4}{s^2 + 4s + 3} = 2 \frac{s + 2}{(s + 1)(s + 3)}$$

Zero : s = -2
Pole s = -1 dan s = -3



Sinyal Eksponensial Kausal dan Anti Kausal

$$x(t) = e^{-at}u(t)$$

$$X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}\,dt = \int_{0^+}^{\infty} e^{-(s+a)t}\,dt$$

$$= -\frac{1}{s+a}e^{-(s+a)t}\Big|_{0^+}^{\infty} = \frac{1}{s+a} \qquad \mathrm{Re}(s) > -a$$

$$x(t) = -e^{-at}u(-t)$$

$$X(s) = \frac{1}{s+a} \qquad \mathrm{Re}(s) < -a$$

$x(t)$	$X(s)$	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}(s) > 0$
$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}(s) < 0$
$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}(s) > 0$
$t^k u(t)$	$\frac{k!}{s^{k+1}}$	$\mathrm{Re}(s) > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}(s) < -\mathrm{Re}(a)$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$

$x(t)$	$X(s)$	ROC
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\text{Re}(s) < -\text{Re}(a)$
$\cos \omega_0 t u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
$\sin \omega_0 t u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
$e^{-at} \cos \omega_0 t u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -\text{Re}(a)$
$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -\text{Re}(a)$

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$	$R' \supset R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	$R' = R$
Shifting in s	$e^{s_0t}x(t)$	$X(s-s_0)$	$R' = R + \mathrm{Re}(s_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(s)$	$R' = aR$
Time reversal	$x(-t)$	$X(-s)$	$R' = -R$
Differentiation in t	$\frac{dx(t)}{dt}$	$sX(s)$	$R' \supset R$
Differentiation in s	$-tx(t)$	$\frac{dX(s)}{ds}$	$R' = R$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$	$R' \supset R \cap \{\mathrm{Re}(s) > 0\}$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$

Daerah Konvergensi

Daerah Konvergensi (DK) / Region of Convergence (ROC)
merupakan rentang nilai agar Transformasi Laplace Konvergen.

$$x(t) = -e^{-at}u(-t)$$

$$X(s) = -\int_{-\infty}^{\infty} e^{-at}u(-t)e^{-st}\,dt = -\int_{-\infty}^{0^-} e^{-(s+a)t}\,dt$$

$$= \frac{1}{s+a}e^{-(s+a)t}\Big|_{-\infty}^{0-} = \frac{1}{s+a} \qquad \mathrm{Re}(s) < -a$$

$$-e^{-at}u(-t) \overset{\text{TL}}{\longleftrightarrow} \frac{1}{s+a} \qquad \mathrm{Re}(s) < -a$$ **ROC**

$$x(t) = e^{at}u(-t)$$

$$X(s) = \int_{-\infty}^{\infty} e^{at}u(-t)e^{-st}\,dt = \int_{-\infty}^{0^-} e^{-(s-a)t}\,dt$$

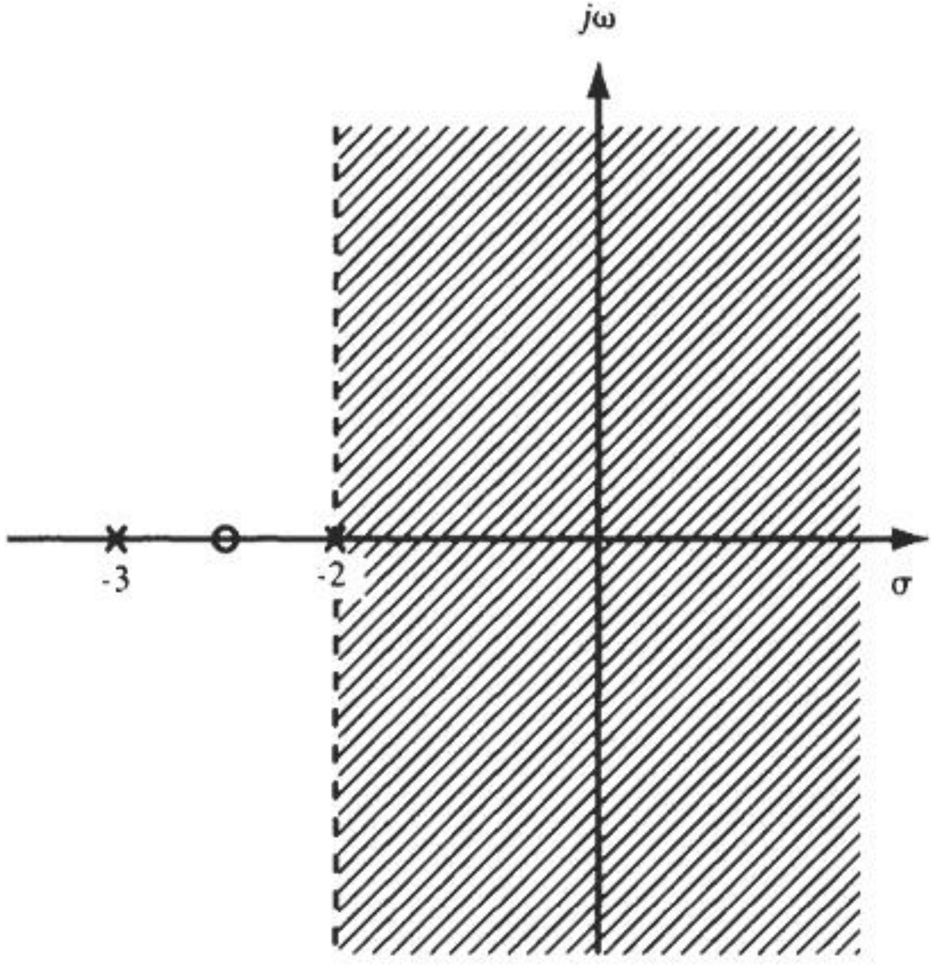
$$= -\frac{1}{s-a}e^{-(s-a)t}\Big|_{-\infty}^{0^-} = -\frac{1}{s-a} \qquad \mathrm{Re}(s) < a$$

$$e^{at}u(-t) \overset{\text{TL}}{\longleftrightarrow} -\frac{1}{s-a} \qquad \mathrm{Re}(s) < a$$ **ROC**

$$x(t) = e^{-2t}u(t) + e^{-3t}u(t)$$

$$e^{-2t}u(t) \overset{\text{TL}}{\longleftrightarrow} \frac{1}{s+2} \qquad \mathrm{Re}(s) > -2$$

$$e^{-3t}u(t) \overset{\text{TL}}{\longleftrightarrow} \frac{1}{s+3} \qquad \mathrm{Re}(s) > -3$$



Transformasi Laplace

$$X(s) = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2(s+\frac{5}{2})}{(s+2)(s+3)}$$

$$\mathrm{Re}(s) > -2$$

Zero : s = $-\frac{5}{2}$

Pole : s = -3 dan s = 2

ROC

$$x(t) = e^{-3t}u(t) + e^{2t}u(-t)$$

$$e^{-3t}u(t) \overset{\text{TL}}{\longleftrightarrow} \frac{1}{s+3} \qquad \text{Re}(s) > -3$$

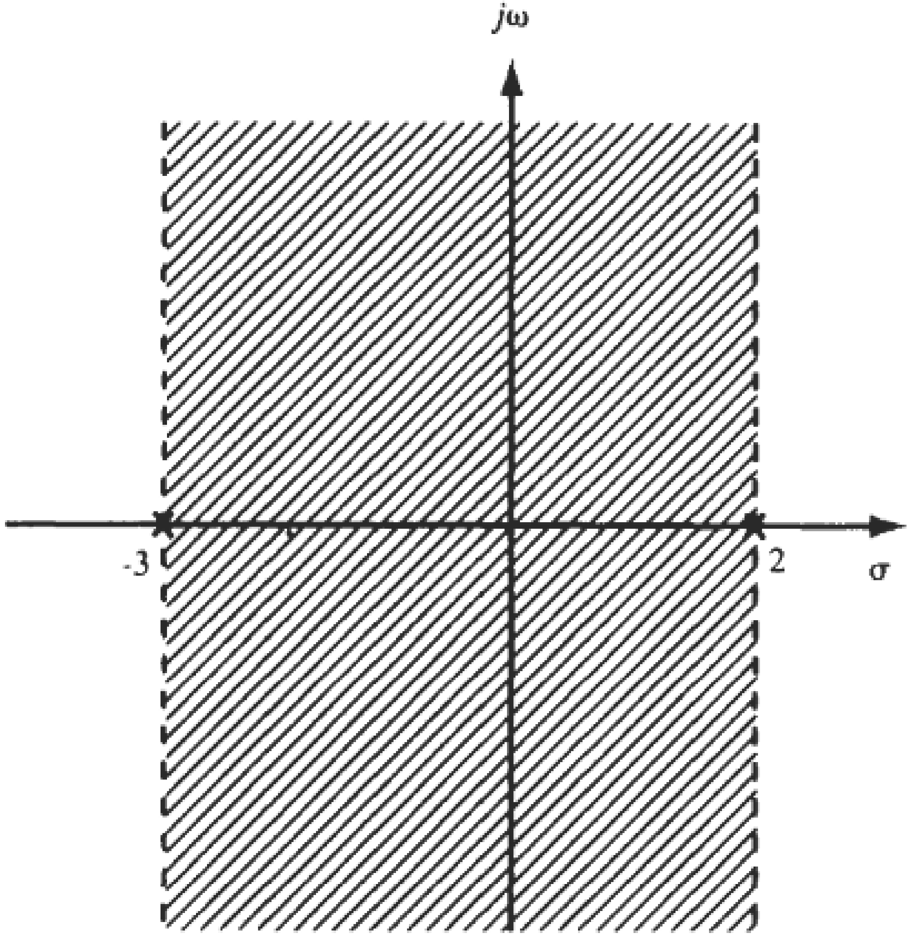
$$e^{2t}u(-t) \overset{\text{TL}}{\longleftrightarrow} -\frac{1}{s-2} \qquad \text{Re}(s) < 2$$

Transformasi Laplace

$$X(s) = \frac{1}{s+3} - \frac{1}{s-2} = \frac{-5}{(s-2)(s+3)}$$

Tidak ada Zero

Pole : s = -3 dan s = 2



$$-3 < \text{Re}(s) < 2$$

ROC

$$x(t) = e^{2t}u(t) + e^{-3t}u(-t)$$

$$e^{2t}u(t) \longleftrightarrow \frac{1}{s-2} \qquad \mathrm{Re}(s) > 2$$

$$e^{-3t}u(-t) \longleftrightarrow -\frac{1}{s+3} \qquad \mathrm{Re}(s) < -3$$

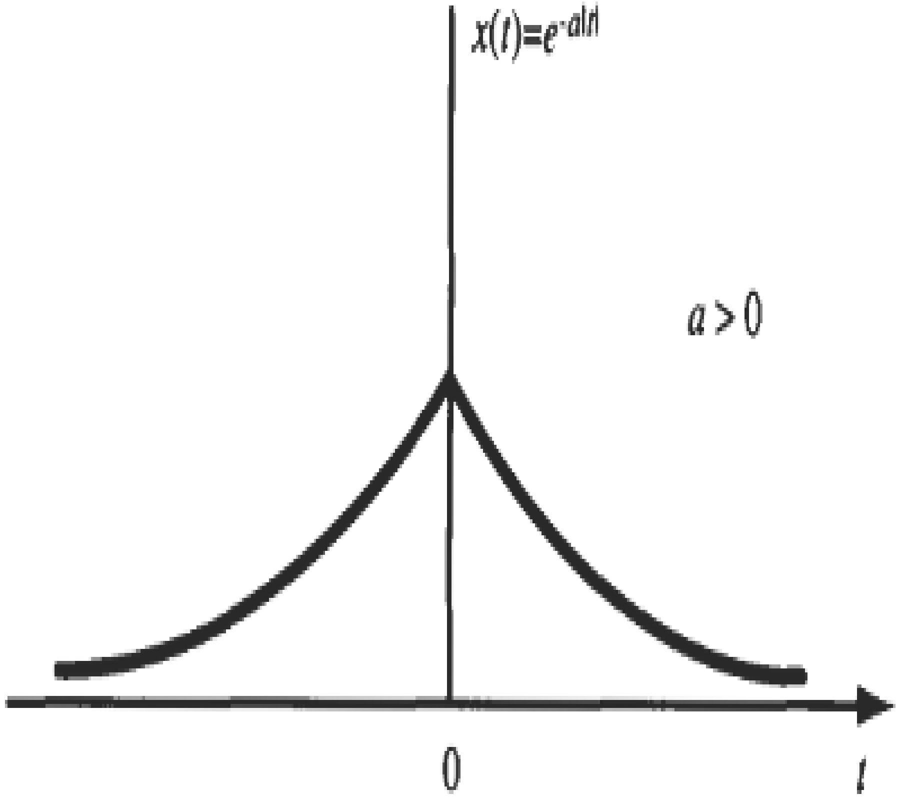
Daerah Konvergensi tidak saling beririsan, artinya sinyal ini tidak mempunyai Transformasi Laplace

$$x(t) = e^{-a|t|}$$

$$x(t) = e^{-at}u(t) + e^{at}u(-t)$$

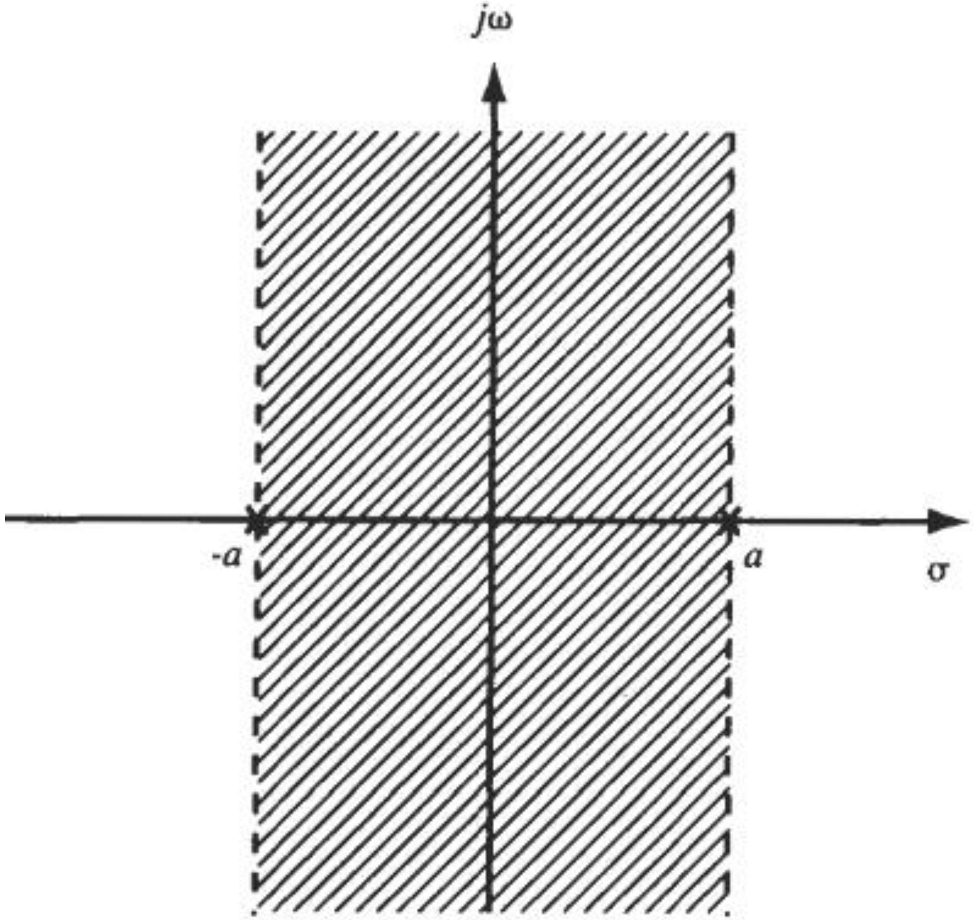
$$e^{-at}u(t) \overset{\text{TL}}{\longleftrightarrow} \frac{1}{s+a} \qquad \mathrm{Re}(s) > -a$$

$$e^{at}u(-t) \overset{\text{TL}}{\longleftrightarrow} -\frac{1}{s-a} \qquad \mathrm{Re}(s) < a$$



$$X(s) = \frac{1}{s+a} - \frac{1}{s-a} = \frac{-2a}{s^2 - a^2}$$

$-a < \text{Re}(s) < a$ **ROC**



Tidak ada Zero

Pole : s = -a dan s = a

$$x(t) = \delta(t - t_0)$$

Transformasi Laplace Pergeseran Waktu Sinyal Impuls Satuan (Time Shifting)

$$\delta(t - t_0) \overset{\text{TL}}{\longleftrightarrow} e^{-st_0} \qquad \text{all } s \qquad \text{ROC}$$

$$x(t) = u(t - t_0)$$

Transformasi Laplace Pergeseran Waktu Sinyal Unit Step (Time Shifting)

$$u(t - t_0) \overset{\text{TL}}{\longleftrightarrow} \frac{e^{-st_0}}{s} \qquad \text{Re}(s) > 0 \qquad \text{ROC}$$

$$x(t) = e^{-2t}[u(t) - u(t-5)]$$

$$x(t) = e^{-2t}[u(t) - u(t-5)] = e^{-2t}u(t) - e^{-2t}u(t-5)$$

$$= e^{-2t}u(t) - e^{-10}e^{-2(t-5)}u(t-5)$$

Transformasi Laplace

$$X(s) = \frac{1}{s+2} - e^{-10}e^{-5s}\frac{1}{s+2} = \frac{1}{s+2}\left(1 - e^{-5(s+2)}\right) \qquad \mathrm{Re}(s) > -2$$

ROC

$$x(t) = \sum_{k=0}^{\infty} \delta(t - kT)$$

Transformasi Laplace Pergeseran Waktu Penumlahan Deretan Sinyal Impuls Satuan dengan Perioda T (Time Shifting)

$$X(s) = \sum_{k=0}^{\infty} e^{-skT} = \sum_{k=0}^{\infty} (e^{-sT})^k = \frac{1}{1 - e^{-sT}} \qquad \operatorname{Re}(s) > 0$$

ROC

$$x(t) = \delta(at + b)$$

$$f(t) = \delta(at) \longleftrightarrow F(s) = \frac{1}{|a|} \qquad \text{all } s$$

$$x(t) = \delta(at + b) = \delta\left[a\left(t + \frac{b}{a}\right)\right] = f\left(t + \frac{b}{a}\right)$$

$$X(s) = e^{sb/a} F(s) = \frac{1}{|a|} e^{sb/a} \qquad \text{all } s$$

ROC

Transformasi Laplace Invers

$$x(t) = \mathscr{L}^{-1}\{X(s)\}$$

$$x(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}\,ds$$

$$X(s) = \frac{N(s)}{D(s)} = k\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

$$X(s) = \frac{c_1}{s-p_1} + \cdots + \frac{c_n}{s-p_n}$$

$$c_k = (s-p_k)X(s)\big|_{s=p_k}$$

Bentuk Irreducible (Pangkat Penyebut D(s) Lebih Tinggi dari Pembilang N(s))

$$X(s) = \frac{N(s)}{D(s)} = Q(s) + \frac{R(s)}{D(s)}$$

Bentuk Reducible (Pangkat Pembilang Lebih Tinggi dari Penyebut)

Transformasi Laplace Invers

$$x(t) = e^{-t}u(t)$$

$$(a) \quad X(s) = \frac{1}{s+1}, \ \mathrm{Re}(s) > -1$$

$$(b) \quad X(s) = \frac{1}{s+1}, \ \mathrm{Re}(s) < -1$$

$$x(t) = -e^{-t}u(-t)$$

$$(c) \quad X(s) = \frac{s}{s^2+4}, \ \mathrm{Re}(s) > 0$$

$$x(t) = \cos 2t u(t)$$

$$(d) \quad X(s) = \frac{s+1}{(s+1)^2+4}, \ \mathrm{Re}(s) > -1$$

$$x(t) = e^{-t} \cos 2t u(t)$$

(a) $X(s) = \dfrac{2s+4}{s^2+4s+3}$, $\mathrm{Re}(s) > -1$

(b) $X(s) = \dfrac{2s+4}{s^2+4s+3}$, $\mathrm{Re}(s) < -3$

(c) $X(s) = \dfrac{2s+4}{s^2+4s+3}$, $-3 < \mathrm{Re}(s) < -1$

$$X(s) = \frac{2s+4}{s^2+4s+3} = 2\frac{s+2}{(s+1)(s+3)} = \frac{c_1}{s+1} + \frac{c_2}{s+3}$$

$$c_1 = (s+1)X(s)|_{s=-1} = 2\frac{s+2}{s+3}\bigg|_{s=-1} = 1$$

$$c_2 = (s+3)X(s)|_{s=-3} = 2\frac{s+2}{s+1}\bigg|_{s=-3} = 1$$

$$X(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$$X(s) = \frac{2s+4}{s^2+4s+3}, \ \mathrm{Re}(s) > -1 \qquad X(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$$x(t) = e^{-t}u(t) + e^{-3t}u(t) = (e^{-t} + e^{-3t})u(t)$$

$$X(s) = \frac{2s+4}{s^2+4s+3}, \ \mathrm{Re}(s) < -3 \qquad X(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$$x(t) = -e^{-t}u(-t) - e^{-3t}u(-t) = -(e^{-t} + e^{-3t})u(-t)$$

$$X(s) = \frac{2s+4}{s^2+4s+3}, \ -3 < \mathrm{Re}(s) < -1$$

$$x(t) = -e^{-t}u(-t) + e^{-3t}u(t)$$

$$X(s) = \frac{5s + 13}{s(s^2 + 4s + 13)} \qquad \mathrm{Re}(s) > 0$$

$$X(s) = \frac{5s + 13}{s(s^2 + 4s + 13)} = \frac{5s + 13}{s(s + 2 - j3)(s + 2 + j3)}$$

$$= \frac{c_1}{s} + \frac{c_2}{s - (-2 + j3)} + \frac{c_3}{s - (-2 - j3)}$$

$$c_1 = sX(s)\big|_{s=0} = \frac{5s + 13}{s^2 + 4s + 13}\bigg|_{s=0} = 1$$

$$c_2 = (s + 2 - j3)X(s)\big|_{s=-2+j3} = \frac{5s + 13}{s(s + 2 + j3)}\bigg|_{s=-2+j3} = -\frac{1}{2}(1 + j)$$

$$c_3 = (s + 2 + j3)X(s)\big|_{s=-2-j3} = \frac{5s + 13}{s(s + 2 - j3)}\bigg|_{s=-2-j3} = -\frac{1}{2}(1 - j)$$

$$X(s) = \frac{1}{s} - \frac{1}{2}(1+j)\frac{1}{s-(-2+j3)} - \frac{1}{2}(1-j)\frac{1}{s-(-2-j3)}$$

$$x(t) = u(t) - \tfrac{1}{2}(1+j)e^{(-2+j3)t}u(t) - \tfrac{1}{2}(1-j)e^{(-2-j3)t}u(t)$$

$$e^{(-2 \pm j3)t} = e^{-2t}e^{\pm j3t} = e^{-2t}(\cos 3t \pm j\sin 3t)$$

Transformasi Laplace Invers

$$\begin{aligned} x(t) &= u(t) - e^{-2t}(\cos 3t - \sin 3t)u(t) \\ &= \left[1 - e^{-2t}(\cos 3t - \sin 3t)\right]u(t) \end{aligned}$$

$$X(s)=\frac{s^2+2s+5}{(s+3)(s+5)^2} \qquad \operatorname{Re}(s)>-3 \qquad X(s)=\frac{c_1}{s+3}+\frac{\lambda_1}{s+5}+\frac{\lambda_2}{(s+5)^2}$$

$$c_1=(s+3)X(s)\big|_{s=-3}=\left.\frac{s^2+2s+5}{(s+5)^2}\right|_{s=-3}=2$$

$$\lambda_2=(s+5)^2X(s)\big|_{s=-5}=\left.\frac{s^2+2s+5}{s+3}\right|_{s=-5}=-10$$

$$\lambda_1=\frac{d}{ds}\left[(s+5)^2X(s)\right]\Big|_{s=-5}=\frac{d}{ds}\left[\frac{s^2+2s+5}{s+3}\right]\Bigg|_{s=-5}$$

$$=\left.\frac{s^2+6s+1}{(s+3)^2}\right|_{s=-5}=-1$$

$$X(s) = \frac{s^2 + 2s + 5}{(s+3)(s+5)^2} \qquad \mathrm{Re}(s) > -3$$

$$X(s) = \frac{2}{s+3} - \frac{1}{s+5} - \frac{10}{(s+5)^2}$$

Transformasi Laplace Invers

$$\begin{aligned} x(t) &= 2e^{-3t}u(t) - e^{-5t}u(t) - 10te^{-5t}u(t) \\ &= [2e^{-3t} - e^{-5t} - 10te^{-5t}]u(t) \end{aligned}$$

$$X(s) = \frac{2 + 2se^{-2s} + 4e^{-4s}}{s^2 + 4s + 3} \qquad \text{Re}(s) > -1$$

$$X(s) = X_1(s) + X_2(s)e^{-2s} + X_3(s)e^{-4s}$$

$$X_1(s) = \frac{2}{s^2 + 4s + 3} \qquad X_2(s) = \frac{2s}{s^2 + 4s + 3} \qquad X_3(s) = \frac{4}{s^2 + 4s + 3}$$

$$x_1(t) \longleftrightarrow X_1(s) \qquad x_2(t) \longleftrightarrow X_2(s) \qquad x_3(t) \longleftrightarrow X_3(s)$$

$$x(t) = x_1(t) + x_2(t-2) + x_3(t-4)$$

$$X_1(s) = \frac{1}{s+1} - \frac{1}{s+3} \longleftrightarrow x_1(t) = (e^{-t} - e^{-3t})u(t)$$

$$X_2(s) = \frac{-1}{s+1} + \frac{3}{s+3} \longleftrightarrow x_2(t) = (-e^{-t} + 3e^{-3t})u(t)$$

$$X_3(s) = \frac{2}{s+1} - \frac{2}{s+3} \longleftrightarrow x_3(t) = 2(e^{-t} - e^{-3t})u(t)$$

Transformasi Laplace Invers

$$x(t) = (e^{-t} - e^{-3t})u(t) + [-e^{-(t-2)} + 3e^{-3(t-2)}]u(t-2)$$

$$+ 2[e^{-(t-4)} - e^{-3(t-4)}]u(t-4)$$

Tentukan Transformasi Laplace Invers dari persamaan berikut :

$$(a) \quad X(s) = \frac{2s+1}{s+2}, \ \mathrm{Re}(s) > -2$$

$$(b) \quad X(s) = \frac{s^2+6s+7}{s^2+3s+2}, \ \mathrm{Re}(s) > -1$$

$$(c) \quad X(s) = \frac{s^3+2s^2+6}{s^2+3s}, \ \mathrm{Re}(s) > 0$$

$$(a)\quad X(s) = \frac{2s+1}{s+2}, \ \mathrm{Re}(s) > -2$$

$$x(t) = 2\delta(t) - 3e^{-2t}u(t)$$

Transformasi Laplace Invers

$$(b)\quad X(s) = \frac{s^2+6s+7}{s^2+3s+2}, \ \mathrm{Re}(s) > -1$$

$$X(s) = \frac{s^2+6s+7}{s^2+3s+2} = 1 + \frac{3s+5}{s^2+3s+2} = 1 + \frac{3s+5}{(s+1)(s+2)}$$

$$X_1(s) = \frac{3s+5}{(s+1)(s+2)} = \frac{c_1}{s+1} + \frac{c_2}{s+2}$$

$$c_1 = (s+1)X_1(s)\big|_{s=-1} = \left.\frac{3s+5}{s+2}\right|_{s=-1} = 2$$

$$c_2 = (s+2)X_1(s)\big|_{s=-2} = \left.\frac{3s+5}{s+1}\right|_{s=-2} = 1$$

$$X(s) = 1 + \frac{2}{s+1} + \frac{1}{s+2} \qquad \mathrm{Re}(s) > -1.$$

Transformasi Laplace Invers

$$x(t) = \delta(t) + (2e^{-t} + e^{-2t})u(t)$$

$$(c)\quad X(s)=\frac{s^3+2s^2+6}{s^2+3s},\ \mathrm{Re}(s)>0$$

$$X(s)=\frac{s^3+2s^2+6}{s^2+3s}=s-1+\frac{3s+6}{s(s+3)}$$

$$X_1(s)=\frac{3s+6}{s(s+3)}=\frac{c_1}{s}+\frac{c_2}{s+3}$$

$$c_1=sX_1(s)\big|_{s=0}=\frac{3s+6}{s+3}\bigg|_{s=0}=2$$

$$c_2=(s+3)X_1(s)\big|_{s=-3}=\frac{3s+6}{s}\bigg|_{s=-3}=1$$

$$X(s)=s-1+\frac{2}{s}+\frac{1}{s+3}$$

Transformasi Laplace Invers

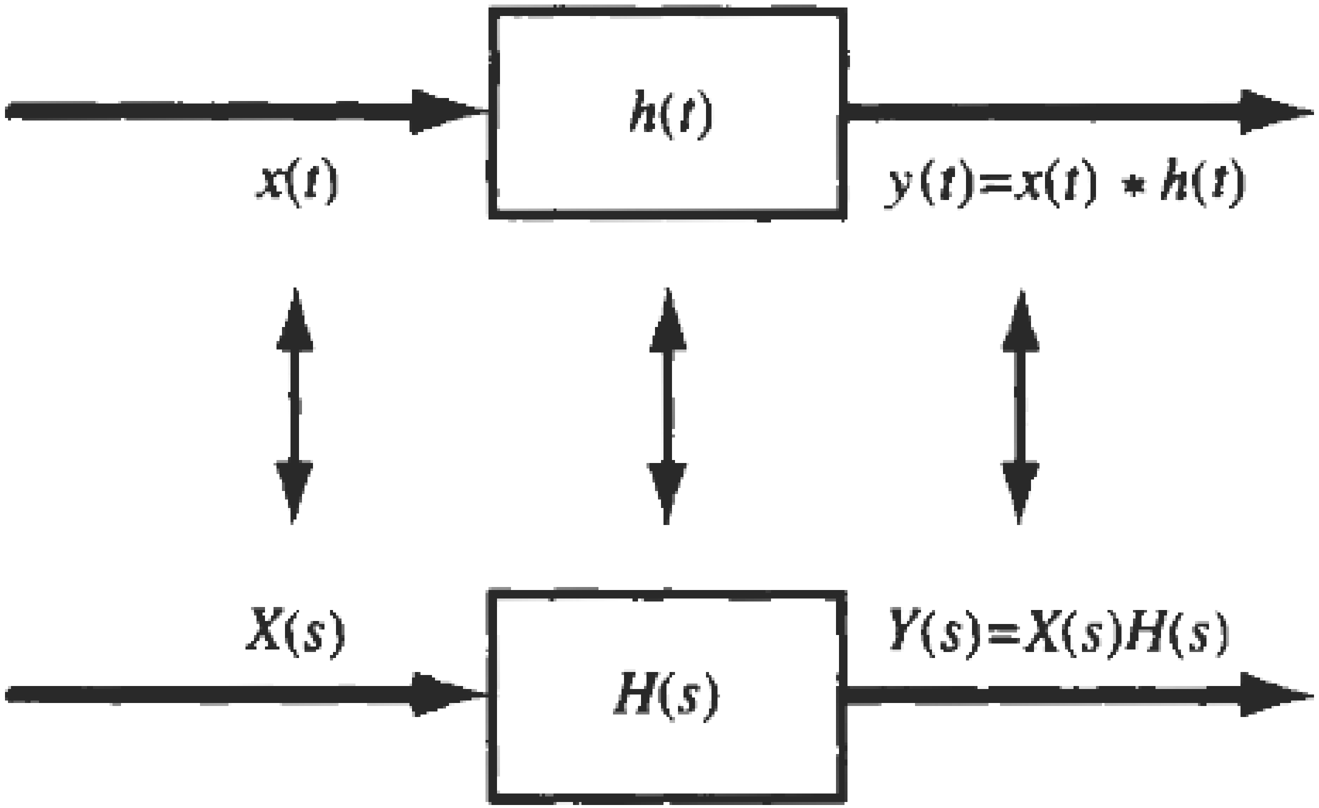
$$x(t)=\delta'(t)-\delta(t)+(2+e^{-3t})u(t)$$

Fungsi Transfer Sistem

Konvolusi di kawasan waktu

$y(t) = x(t) * h(t)$ ⟶ $Y(s) = X(s)H(s)$ **Perkalian di kawasan Frekuensi**

Fungsi Transfer Sistem $H(s) = \dfrac{Y(s)}{X(s)}$



Karakterisasi Sistem

1. Sistem Kausal

Karakteristik : Sinyal sisi kanan, sehingga ROC $\mathrm{Re}(s) > \sigma_{\max}$

2. Sistem Anti Kausal

Karakteristik Sinyal sisi kiri, sehingga ROC $\mathrm{Re}(s) < \sigma_{\min}$

3. Sistem Stabil

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

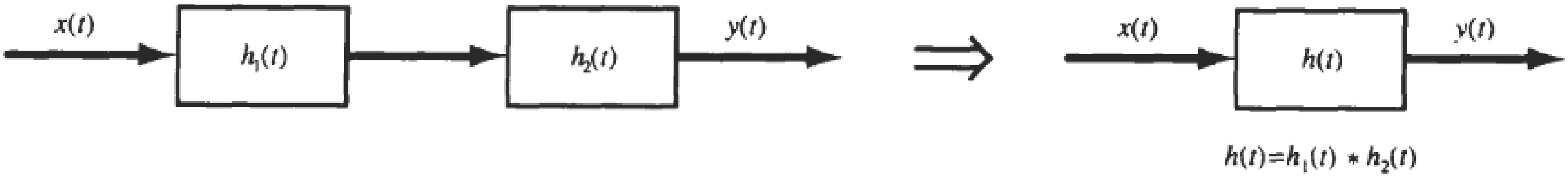
Karakteristik : BIBO (Bounded Input, Bounded Input), Apabila di gambar di bidang s Daerah Konvergensi-nya (ROC) melingkupi sumbu jω

4. Sistem Kausal dan Stabil

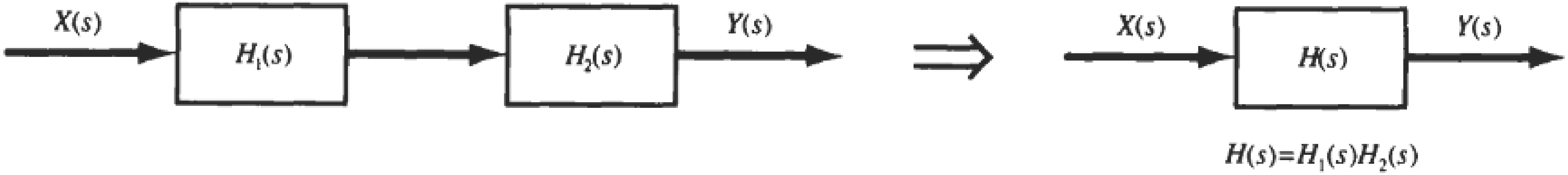
Karakteristik : $\mathrm{Re}(s) > \sigma_{\max}$ dengan $\sigma_{\max} < 0$

Sehingga melingkupi sumbu ***jω***

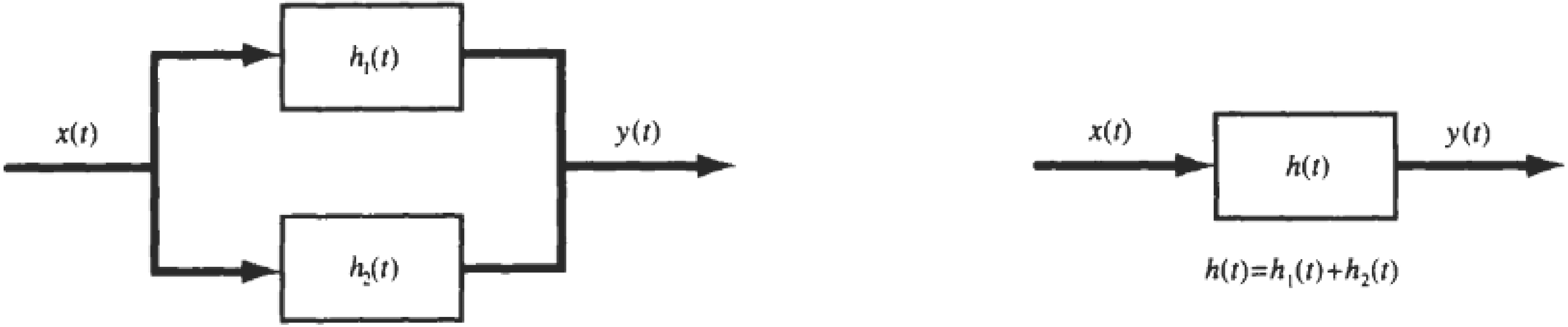
Interkoneksi Sistem



Interkoneksi Sistem Secara Seri (Cascade) di kawasan waktu

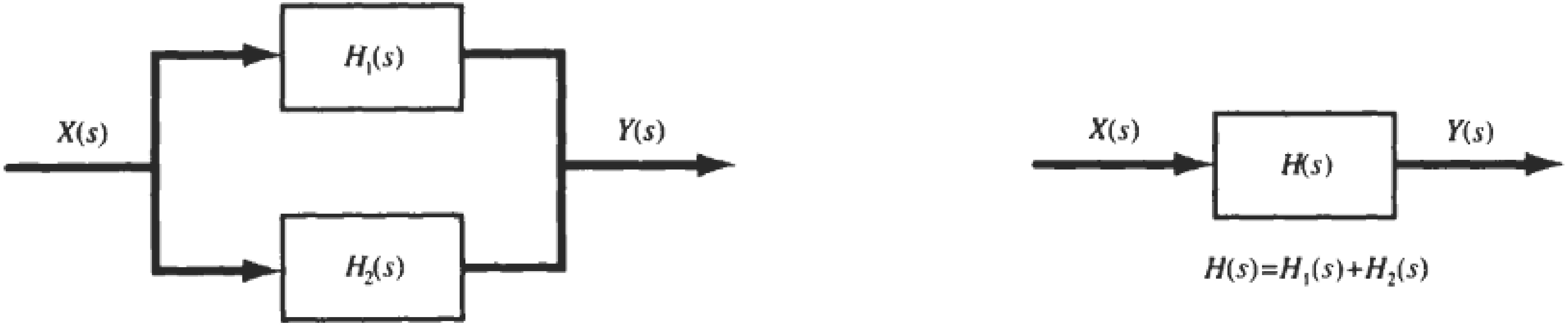


Interkoneksi Sistem Secara Seri (Cascade) di kawasan frekuensi



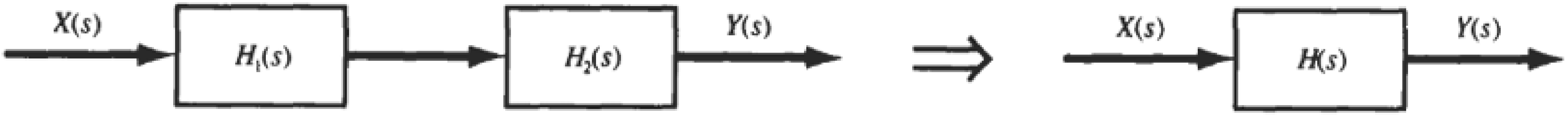
(*a*)

Interkoneksi Sistem Secara Paralel di kawasan waktu



(*b*)

Interkoneksi Sistem Secara Paralel di kawasan frekuensi



$H(s)=H_1(s)H_2(s)$

$$h_1(t) = e^{-2t}u(t) \longleftrightarrow H_1(s) = \frac{1}{s+2} \qquad \mathrm{Re}(s) > -2$$

$$h_2(t) = 2e^{-t}u(t) \longleftrightarrow H_2(s) = \frac{2}{s+1} \qquad \mathrm{Re}(s) > -1$$

$$H(s) = H_1(s)H_2(s) = \frac{2}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{2}{s+2} \qquad \mathrm{Re}(s) > -1$$

$$h(t) = 2(e^{-t} - e^{-2t})u(t)$$

Sistem stabil Karena melingkupi sumbu jω

Misal x(t) Input sistem dan y(t) = output sistem

$$x(t) = u(t),\ y(t) = 2e^{-3t}u(t) \qquad X(s) = \frac{1}{s} \qquad \mathrm{Re}(s) > 0$$

$$Y(s) = X(s)H(s) \qquad Y(s) = \frac{2}{s+3} \qquad \mathrm{Re}(s) > -3$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s}{s+3} \qquad \mathrm{Re}(s) > -3$$ **Fungsi Transfer Sistem**

$$H(s) = \frac{2s}{s+3} = \frac{2(s+3)-6}{s+3} = 2 - \frac{6}{s+3} \qquad \mathrm{Re}(s) > -3$$

$$h(t) = 2\delta(t) - 6e^{-3t}u(t)$$ **Respon Impuls Sistem**

Input Sistem

Fungsi Transfer Sistem

$$x(t) = e^{-t}u(t) \longleftrightarrow \frac{1}{s+1} \qquad \mathrm{Re}(s) > -1 \qquad H(s) = \frac{2s}{s+3}$$

$$Y(s) = X(s)H(s) = \frac{2s}{(s+1)(s+3)} \qquad \mathrm{Re}(s) > -1$$

(Sistem Stabil dan Kausal)

$$Y(s) = -\frac{1}{s+1} + \frac{3}{s+3}$$

Keluaran Sistem

$$y(t) = (-e^{-t} + 3e^{-3t})u(t)$$

Respon Impuls Sistem **Input Sistem**

$$h(t) = e^{-\alpha t}u(t) \qquad x(t) = e^{\alpha t}u(-t) \qquad \alpha > 0$$

Fungsi Transfer Sistem

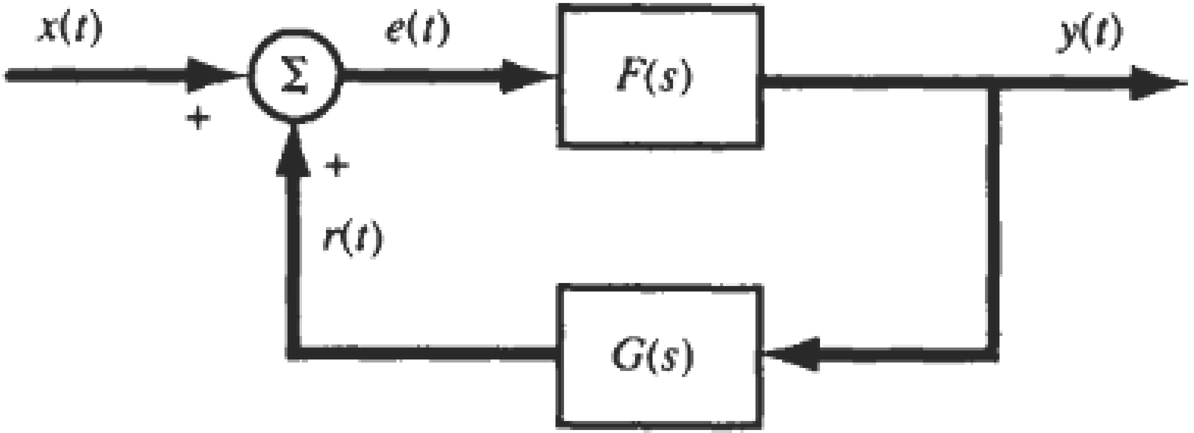
$$H(s) = \frac{1}{s+\alpha} \qquad \text{Re}(s) > -\alpha$$

$$X(s) = -\frac{1}{s-\alpha} \qquad \text{Re}(s) < \alpha$$

$$Y(s) = X(s)H(s) = -\frac{1}{(s+\alpha)(s-\alpha)} = -\frac{1}{s^2-\alpha^2} \qquad -\alpha < \text{Re}(s) < \alpha$$

$$y(t) = \frac{1}{2\alpha}e^{-\alpha|t|}$$

Keluaran Sistem



$$x(t) \longleftrightarrow X(s) \qquad y(t) \longleftrightarrow Y(s) \qquad r(t) \longleftrightarrow R(s) \qquad e(t) \longleftrightarrow E(s)$$

$$Y(s) = E(s)F(s)$$

$$R(s) = Y(s)G(s)$$

$$e(t) = x(t) + r(t)$$

$$E(s) = X(s) + R(s)$$

$$Y(s) = [X(s) + Y(s)G(s)]F(s)$$

$$[1 - F(s)G(s)]Y(s) = F(s)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 - F(s)G(s)}$$

Fungsi Transfer Feedback System

Hubungan input-output suatu sistem LTI dinyatakan dalam persamaan berikut :

$$\frac{dy(t)}{dt} + ay(t) = x(t)$$

$$sY(s) + aY(s) = X(s)$$

$$(s+a)Y(s) = X(s)$$

Fungsi Transfer Sistem H(s)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+a}$$

Respon Impuls Sistem

(Asumsi Sistem Kausal ROC >-a)

$$h(t) = e^{-at}u(t)$$

Hubungan input-output suatu sistem LTI dinyatakan dalam persamaan berikut :

$$y'(t) + 2y(t) = x(t) + x'(t)$$

Tentukan Fungsi Transfer Sistem H(s)

$$sY(s) + 2Y(s) = X(s) + sX(s)$$

$$(s+2)Y(s) = (s+1)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+1}{s+2} = \frac{s+2-1}{s+2} = 1 - \frac{1}{s+2}$$

Respon Impuls Sistem (Asumsi Sistem Kausal)

$$h(t) = \delta(t) - e^{-2t}u(t)$$

Hubungan input-output suatu sistem LTI dinyatakan dalam persamaan berikut :

$$y''(t) + y'(t) - 2y(t) = x(t)$$

Tentukan Fungsi Transfer Sistem H(s)

$$s^2Y(s) + sY(s) - 2Y(s) = X(s)$$

$$(s^2 + s - 2)Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + s - 2} = \frac{1}{(s+2)(s-1)}$$

$$H(s) = \frac{1}{(s+2)(s-1)} = -\frac{1}{3}\frac{1}{s+2} + \frac{1}{3}\frac{1}{s-1}$$

$$H(s) = \frac{1}{(s+2)(s-1)} = -\frac{1}{3}\frac{1}{s+2} + \frac{1}{3}\frac{1}{s-1}$$

Respon Impuls Sistem untuk Sistem Kausal, ROC $\mathrm{Re}(s) > 1$

$$h(t) = -\tfrac{1}{3}(e^{-2t} - e^{t})u(t)$$

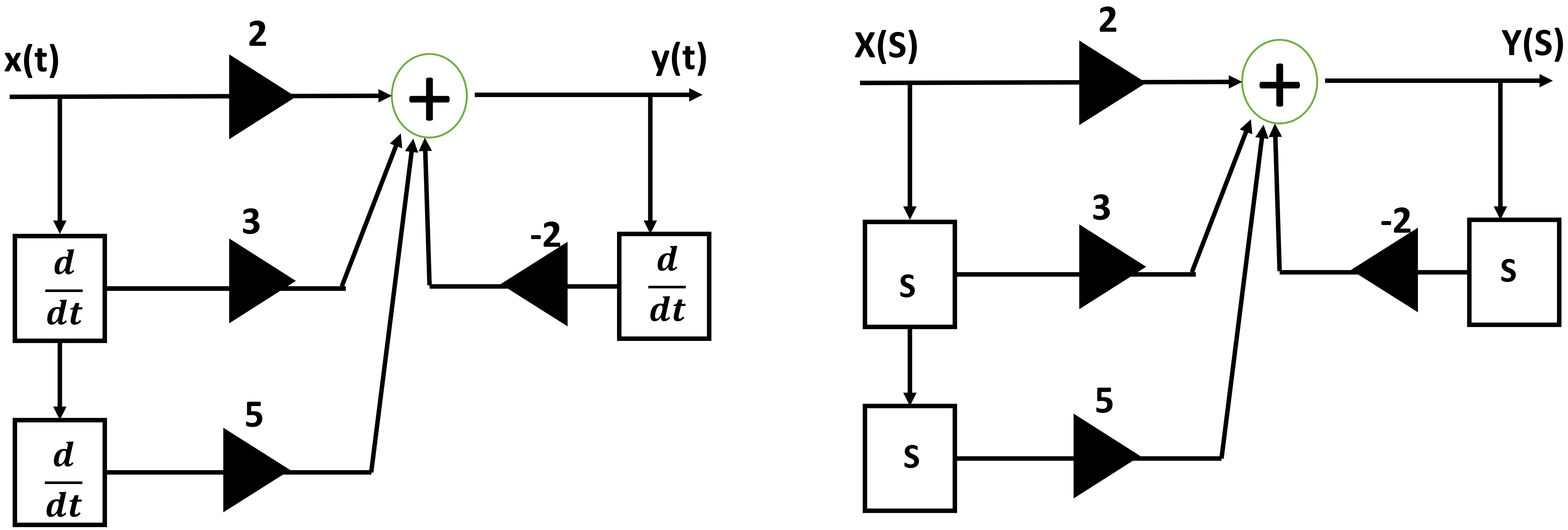
Respon Impuls Sistem untuk Sistem Stabil, ROC $-2 < \mathrm{Re}(s) < 1$

$$h(t) = -\tfrac{1}{3}e^{-2t}u(t) - \tfrac{1}{3}e^{t}u(-t)$$

Respon Impuls Sistem Tidak Kausal dan Tidak Stabil $\mathrm{Re}(s) < -2.$

$$h(t) = \tfrac{1}{3}e^{-2t}u(-t) - \tfrac{1}{3}e^{t}u(-t)$$

Struktur Realisasi Sistem



Persamaan Sistem : Y(S) = 2 X(S) + 3 s X(S) + 5s^{2}X(S) − 2 sY(S)

Y(S)+2sY(S) = 2 X(S) + 3 s X(S) + 5s^2X(S)

(2s+1) Y(S) = (2+3s+5s^2) X(S)

$$\frac{Y(S)}{X(S)} = \frac{2s+1}{2+3s+5s^2}$$ ⟶ Fungsi Transfer H(S)

Gambar Struktur Realisasi Sistem

Diketahui Fungsi Transfer Sistem H(S) = $\frac{\mathbf{5+2}s}{\mathbf{1-2}s}$

$$\frac{Y(S)}{X(S)} = \frac{\mathbf{5+2}s}{\mathbf{1-2}s}$$

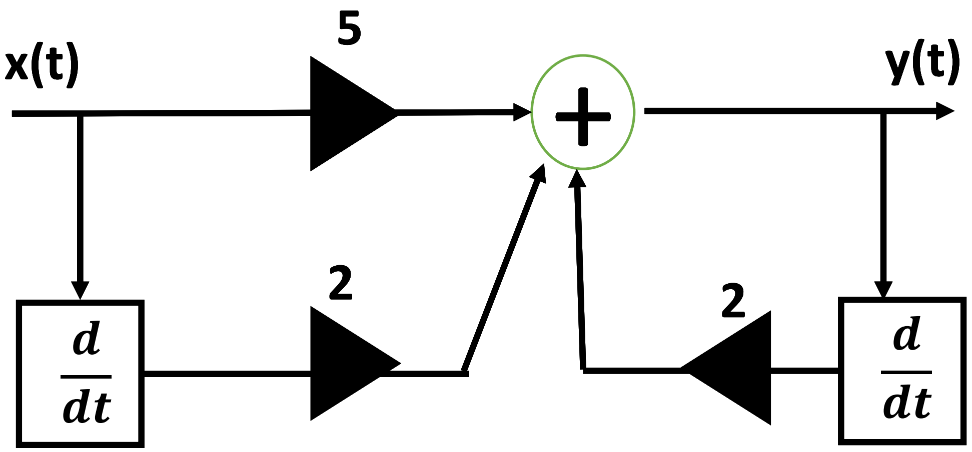
Y(S) = $\frac{\mathbf{5+2}s}{\mathbf{1-2}s}$ X(S)

Y(S)- 2sY(S) = 5X(S) + 2 sX(S)

Y(S) = 5X(S) + 2 sX(S) + 2sY(S)

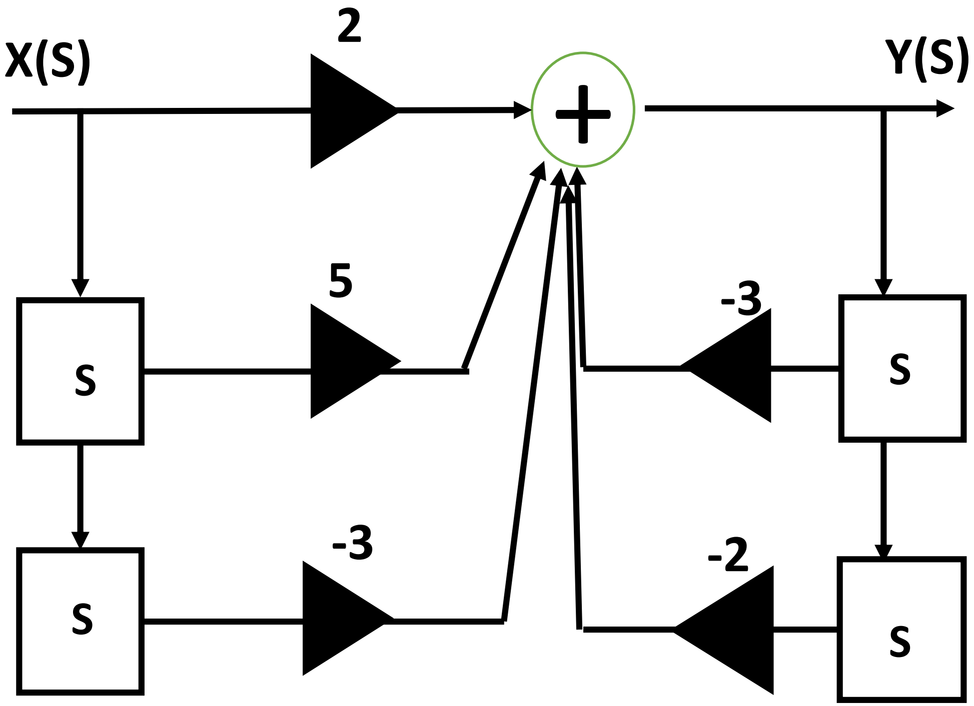
Persamaan differensial

y(t) = 5 x(t) + 2$\frac{d}{dt}x(t) + 2\,\frac{d}{dt}y(t)$



Aturan Mason

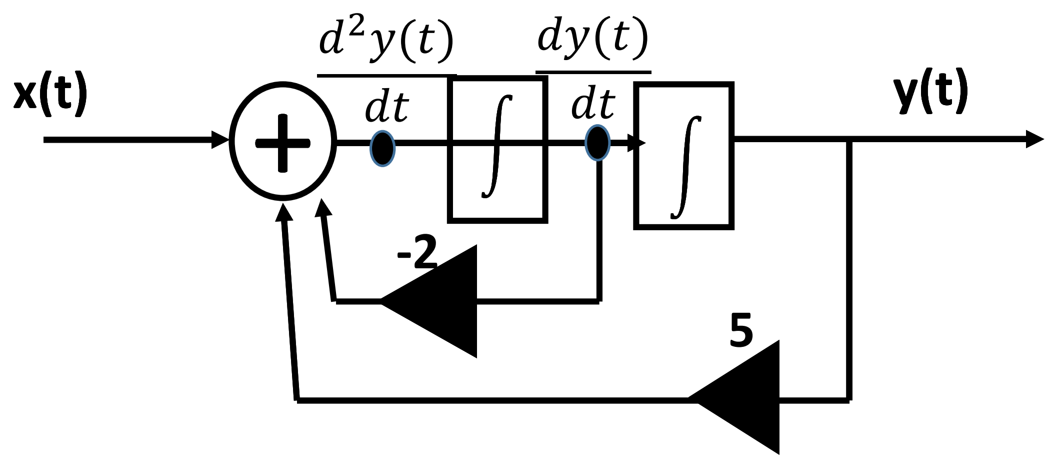
- Fungsi Transfer = $\frac{\sum Forward}{1-\sum Loop}$

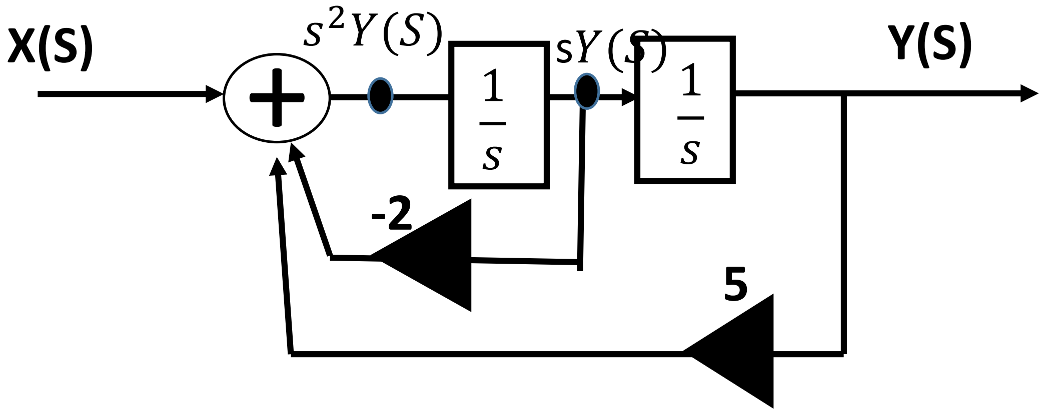


$$H(S) = \frac{2+5s-3s^2}{1-(-3s-2s^2)}$$

$$H(S) = \frac{2+5s-3s^2}{1+3s+2s^2}$$

Tentukan Fungsi Transfer Rangkaian berikut :





$S^2Y(S) = X(S) - 2sY(S) + 5$Y(S)

$S^2Y(S) + 2sY(S) - 5$Y(S) = X(S)

$(S^2+2s - 5)$ Y(S) = X(S)

$\frac{Y(S)}{X(S)} = \frac{1}{(S^2+2s-5)}$

H(S) = $\frac{1}{(S^2+2s-5)}$

H(S) = $\frac{\frac{1}{s}\frac{1}{s}}{1-(\frac{-2}{s}+\frac{5}{s^2})}$

H(S) = $\frac{\frac{1}{s^2}}{\frac{s^2+2s-5}{s^2}}$

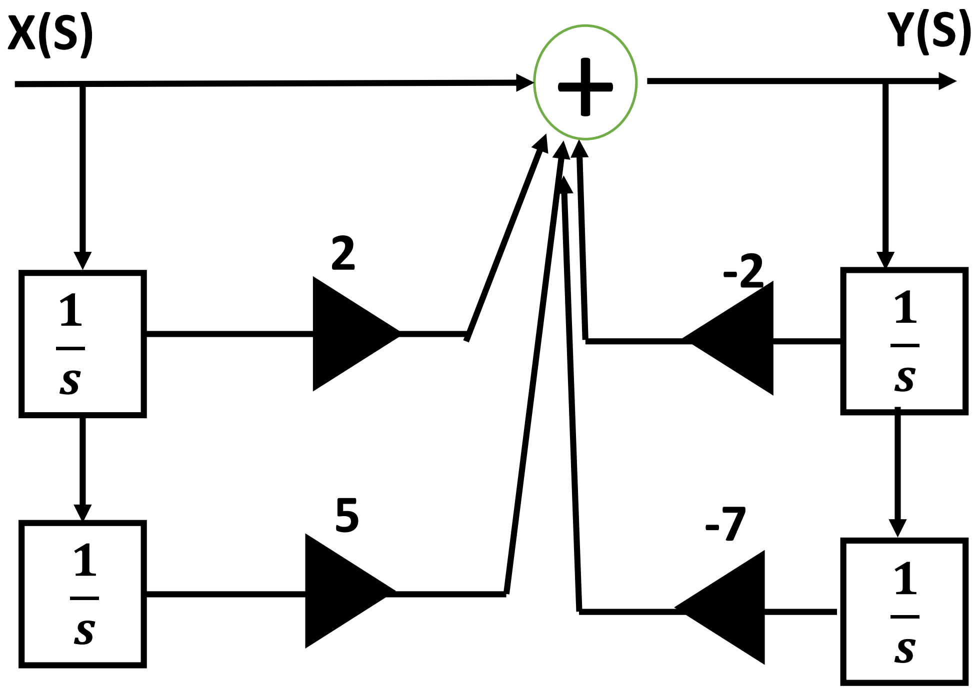
H(S) = $\frac{1}{S^2+2s-5}$

Realisasikan Fungsi Transfer tersebut dengan integrator

$$H(S) = \frac{2s+5}{s^2+2s+7}$$

$$H(S) = \frac{\frac{2}{s}+\frac{5}{s^2}}{1+\frac{2}{s}+\frac{7}{s^2}}$$

$$H(S) = \frac{\frac{2}{s}+\frac{5}{s^2}}{1-(-\frac{2}{s}-\frac{7}{s^2})}$$



Suatu sistem kausal LTI memiliki respon impuls h(t) = $(5e^{-2t} + 2e^{-t})u(t)$

Tentukan :

a. Fungsi Transfer H(S)?

b. Persamaan differensial?

c. Struktur realisasi sistem?

d. Pole dan zero sistem?

e. Apakah sistem stabil?

Jawab

a. H(S) = $\frac{5}{s+2} + \frac{2}{s+1} = \frac{5(s+1)+2(s+2)}{(s+2)(s+1)} = \frac{7s+9}{s^2+3s+2}$

b. $\frac{Y(S)}{X(S)} = \frac{7s+9}{s^2+3s+2}$

$(s^2 + 3s + 2)\ \mathrm{Y(S)} = (7\mathrm{s} + 9)\mathrm{X(S)}$

$s^2Y(S) + 3sY(S) + 2Y(S) = 7\mathrm{sX(S)} + 9\mathrm{X(S)}$

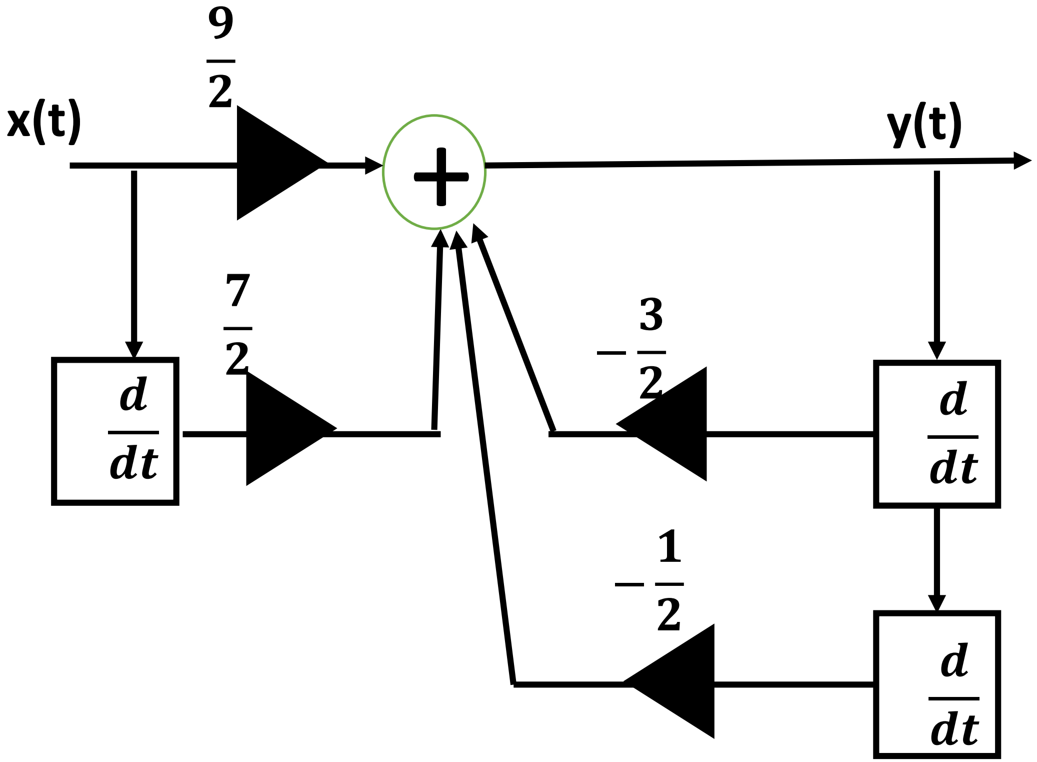
2Y(S) $= 7s\mathrm{X(S)} + 9\mathrm{X(S)} - s^2Y(S) - 3sY(S)$

$$Y(S) = \frac{7}{2} sX(S) + \frac{9}{2} X(S) - \frac{1}{2} s^2 Y(S) - \frac{3}{2} sY(S)$$

Persamaan differensial

$$y(t) = \frac{7}{2}\frac{d}{dt}x(t) + \frac{9}{2}x(t) - \frac{1}{2}\frac{d^2}{dt}y(t) - \frac{3}{2}\frac{d}{dt}y(t)$$

c. Struktur Realisasi Sistem



d. Pole dan Zero Sistem dengan h(t) = $(5e^{-2t} + 2e^{-t})u(t)$

H(S) = $\frac{7s+9}{s^2+3s+2}$

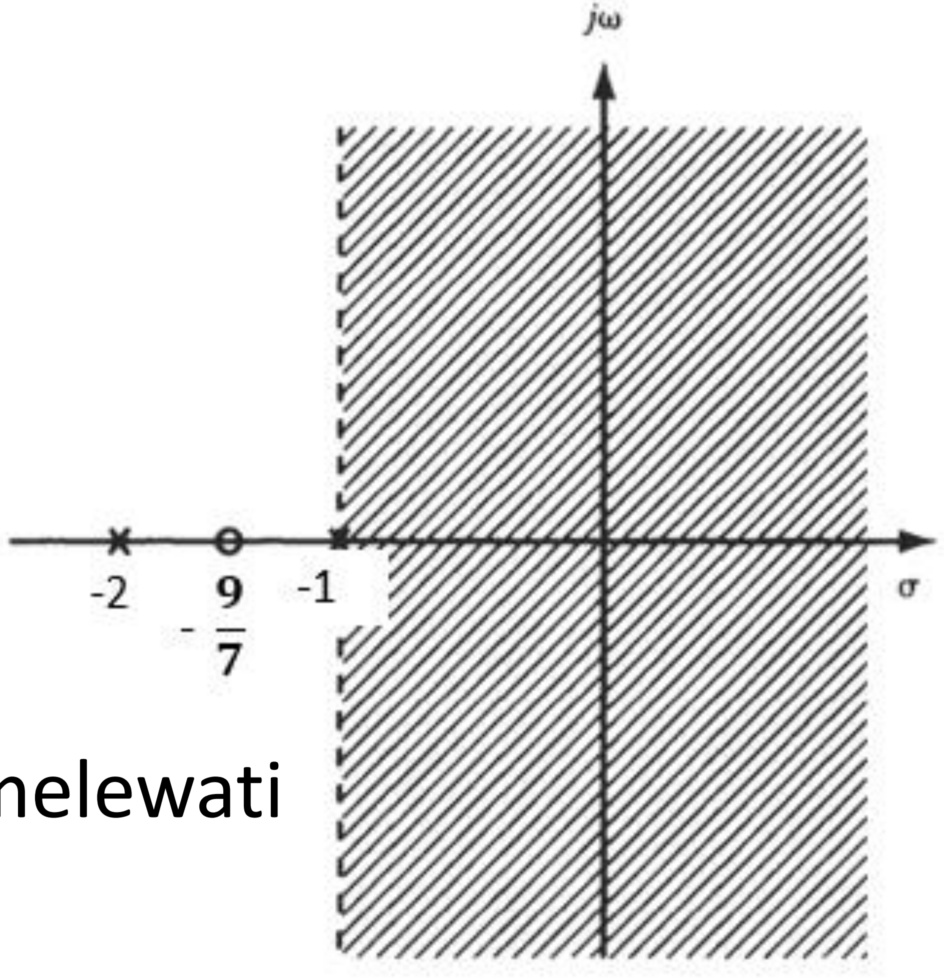
H(S) = $\frac{7s+9}{(s+1)(s+2)}$

Zero : s= $-\frac{\mathbf{9}}{\mathbf{7}}$

Pole : s=-1 dan s = -2

e. Sistem Stabil

Derah Konvergensi ROC melewati sumbu jω



$\mathrm{Re}(s) > -1.$

Sebuah Sistem Waktu Kontinyu dinyatakan oleh persamaan differential :

$y''(t) + 4\,y'(t) + 13\,y(t) = x'(t) + 4x(t)$; $x(t)$ = input ; $y(t)$ = output

a. Cari Fungsi Transfer Sistem $H(s)$!
b. Cari respon impuls $h(t)$!
c. Gambarkan realisasi Sistem !
d. Apakah sistem tersebut stabil ? Jelaskan !
e. Apabilal input $x(t) = 2\delta(t-1)$, Carilah output Sistem $y(t)$!